Main comments: The authors study the index of the half-BPS giant gravitons. While it has been already given in the literature, they propose an alternative derivation from the bulk string theory. They claim that as the effective Lagrangian of the giant gravitons obtained from the D3-brane action can be viewed as the 1d supersymmetric quantum mechanical Lagrangian for the Landau problem describing a particles in a two-dimensional plane with a constant magnetic flux, the index can be evaluated as its Witten index which has contributions from the lowest Landau level. Though the result of the index is already known, their approach would be potentially interesting and useful for experts working on the holographic duality in the context of string theory and supersymmetric gauge theory. However, there are several parts with which I am confused in the draft. I would like the authors to carefully improve the points which I list below.

- While they claim that the index is evaluated from the functional integral, the actual computations seem to be mainly performed in the Hamiltonian formalism rather than the Lagrangian formalism. Although they begin with eq.(3.1) as the bulk index, what is the status of this object? It contains the operator R which is acceptable in the Hamiltonian formalism whereas it also contains the action. The space on which the operator acts is unclear. From the functional integral approach in gauge theory, the index is calculated as a certain partition function on $S^1 \times S^3$ of 4d gauge theory with the parameter q being associated with the radii of compact geometries and adjoint mass parameter. But what does q imply in eq.(3.1) from the bulk approach? Also the sum may not be convergent if the action is not negative definite. It would be better to explain what eq.(3.1) is and how it is derived from the path integral for the bulk theory.

- The integration over μ, the moduli of m branes, introduced in eq.(3.1) has been performed to get the factorized form in eq.(3.16). But I cannot see what μ concretely implies and how and why the integration simply yields the multi-particle index $I_{\text{sugra}}(q)$. If we calculate it from the gravity side by enumerating the fluctuation modes of gas of supergravitons on the $AdS_5 \times S^5$, we will get the single particle index rather than the multi-particle index.

- It is unclear why the correct result can be simply obtained by supersymmetrizing the partial bosonic (gauge fixed) Lagrangian without the $\vec{A}_2$ term as the "doubled theory" with four fermions as discussed in section 3.2. The effective Lagrangian should be derived by carefully reducing the supersymmetric action of D3-brane involving the fermionic terms and taking the kappa symmetry projection to 1d. In general, there will be non-trivial interaction terms, say between the bosonic and fermionic fields in the 1d theory. But such analysis is missing in the draft. Since the derivation of the effective Lagrangian will be the crucial part to correctly reproduce the index from the bulk theory, it would be rather important to explicitly derive the 1d supersymmetric Lagrangian.

- To explain the negative sign, the authors suddenly introduce the current eq.(3.41) constructed by worldvolume fermions. It will be better to make discussion more transparent by considering the fermionic terms in the brane action as I indicated in the previous point.

There are further minor comments.

1. On page 9, “in these coordinate” should be “in these coordinates”.

2. In several occasions, e.g. eq.(2.41), (3.25), L is introduced as a momentum operator, but it may cause a confusion with the radius of AdS_5 for which it was firstly used.

3. While μ is used for mass of particle in eq.(2.31), it again appears as moduli in eq.(3.1) as the moduli of m branes. If they are not related with each other, it would be better to use a different notation.

4. On page 13, it is mentioned “this is the 2d chiral $\mathcal{N} = 4$ supersymmetry algebra”. Why is the supersymmetry algebra associated with 2d spacetime? The effective Lagrangian is 1d quantum mechanics (rather than 2d field theory) whose fields only depend on time coordinate. On the other hand, if it is $\mathcal{N} = (4, 4)$ discussed below, it will be non-chiral supersymmetry rather than chiral supersymmetry.