

## A. Questions/Clarifications

- The authors relaxed the condition of regularity and obtained several new models. I understand that this is one of the highlights of their procedure, given that this type of model, was not achievable via previous methods (with few exceptions). In my understanding, however, the method is also perfectly applicable to find regular R-matrices. Have the authors tried to apply it in such a case and compare the results with the ones in the literature (for example reference [26])? If not, I am not asking them to do that now, but it would have been in principle an extra good check of the completeness of their method. It would be useful to add at least a comment, mentioning if the procedure also applies to classify regular integrable models.
- The concept of Baxterization was mentioned twice but not defined. Could the authors add a sentence or a reference about it? Some readers interested in your models, but with a different research background would perhaps not be familiar with the term.
- A comparison with the literature would be interesting. In particular, some non-regular R-matrices exist in the context of AdS<sub>2</sub>. See solutions 2 to 5 in arXiv:1706.02634v2 [hep-th] for example. The solutions there are graded, but by just changing a few signs they become solutions of the standard YBE. Solution 3 in the mentioned paper, in particular, looks like a graded version of your solution 2 in section 3.3.2. I would encourage the authors to investigate if some of the models are connected by grading and basis transformations.
- I think the procedure developed in this paper could be in principle adapted to, given an R-matrix, classify all Lax matrices satisfying the RLL relations

$$R_{ab}(u-v)L_{aj}(u)L_{bj}(v) = L_{bj}(v)L_{aj}(u)R_{ab}(u-v). \quad (0.1)$$

Also, it could be adapted, such that for a given R-matrix, it would find the solutions of the boundary YBE

$$R_{12}(u-v)K_1(u)R_{21}(u+v)K_2(v) = K_2(v)R_{12}(u+v)K_1(u)R_{21}(u-v). \quad (0.2)$$

Do the authors agree? If so, these applications could also be added to their Conclusions and Outlook section.

## B. Suggestions

The diagrams in Figure 1 and Figure 3 are already really helpful to understand the structure of the authors' procedure. I think, however, that two small additions would make them even clearer:

- In Figure 1, it would be helpful if the authors could add which Stage is being addressed in each box.
- In the same spirit, if the authors could specify in Figure 3 which algorithm from appendix A.2. is used in each box/step, it would make the details of the procedure even clearer to the reader.

## C. Typos:

- Second line of equation (10), there is a  $\partial_\nu$  missing in  $R_{13}(u+v)$ .
- In equation (11) there is a  $\partial_\nu$  missing in  $R_{13}(u+v)$  in both the first and second line.

- In Table 1, if I haven't make any mistakes, H21 (or alternatively H22=H2x) does not satisfy YBE for generic  $k$ . It only satisfies YBE if  $k = 1$  or  $k = -pq$ . Can you please check it?
- In Figure 1, in the box before "End" I would replace "it exists" by "they exist".
- In the paragraph just before section 4.2, "which has"  $\rightarrow$  "which have", and "is shifted in"  $\rightarrow$  "moved to".
- In section 4.2, in "Initialisation", second bullet point: the word "under" is repeated.
- In A.1 `tmpEqnsList[i]` and `doVertList[i]` are defined twice.
- In A.1, algorithm 8, line 6, did you mean "Remove zeros from `dRRRList[i]`" instead of "`algRRRList[i]`"?
- In the description of Algorithm 10, "solling"  $\rightarrow$  "solving".
- In B.2, some solutions are exactly the same. For example  $sol18 = sol19$  and  $sol27 = sol30 = sol33$ , and  $sol29 = sol32 = sol35$ .

#### D. Some possible references:

In footnote 1, the authors ask to be informed of other classification works on the YBE. First of all, let me say that the authors apparently made a choice of citing only references focused on classifications of  $4 \times 4$  R-matrices (but not the ones with higher dimensional Hilbert spaces). I believe that this is an acceptable choice and if that is the case they have (from my knowledge) a complete list. In such a case please disregard all the suggestions below.

For higher dimensional R-matrices some classification works that could be mentioned are

- M. Jimbo, "Quantum R Matrix for the Generalized Toda System," Commun. Math. Phys. 102 (1986) 537–547.
- M. Idzumi, T. Tokihiro & M. Arai, "Solvable nineteen vertex models and quantum spin chains of spin one", J. Phys. I(France) 4, 1151 (1994).
- Z. Maassarani, "Multiplicity Am models", Eur. Phys. J. B 7, 627 (1999), arXiv:9805009.
- R. Pimenta & M. Martins, "The Yang-Baxter equation for PT invariant nineteen vertex models", J. Phys. A 44, 085205 (2011), arXiv:1010.1274.
- N. Crampé, L. Frappat & E. Ragoucy, "Classification of three-state Hamiltonians solvable by the coordinate Bethe ansatz", J. Phys. A 46, 405001 (2013).
- M. Martins, "Integrable three-state vertex models with weights lying on genus five curves", Nucl. Phys. B 874, 243 (2013), arXiv:1303.4010.
- M. De Leeuw, A. Pribytok, A.L. Retore & P. Ryan, "New integrable 1D models of superconductivity", J. Phys. A 53, 385201 (2020), arXiv:1911.01439.
- M. de Leeuw, R. I. Nepomechie and A.L.Retore, "Flag integrable models and generalized graded algebras," JHEP **06** (2023), 113, arXiv:2210.06495.