Higher-order topology protected by latent crystalline symmetries
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Latent symmetries offer the possibility to determine the physical properties of systems beyond the paradigm of conventional geometric symmetries. They are based on the isospectral reduction techniques in graph theory and provide an intricate link between the reduced effective system that exhibits a (hidden) symmetry and the original system. Cospectrality and Hamiltonian matrix power diagnosis underly this approach and make it constructive. Remarkably, the hidden symmetry on the reduced system can be extended to a full symmetry of the original system, which is however, in general, not of geometrical origin but a 'complicated' combination of the geometrical operation on the reduced system plus a e.g. general orthogonal transform of the remaining degrees of freedom. While latent symmetries have been used in previous works to e.g. design flat bands or explain degeneracies, the present work focuses on the extension of the concept of latent symmetries to higher order topological properties of crystalline lattices. This is a topic of immediate interest in the course of the recent developments moving from 'traditional' topological insulators to higher order ones, where higher order polarizations are quantized and lower dimensional edge states dominate the phenomenology.

The present work represents major progress in terms of further developing latent symmetry concepts to topological band structure. It is a remarkable contribution to the state of the art of the field and should definitely be published in Scipost physics. Its merits stem from the fact that higher order symmetries i.e. multiple discrete rotational ones are conceptually implemented to represent latent symmetries and goes all the way to showing a series of concrete examples for lattices exhibiting arbitrary higher order topology by combining 'simple' models via so-called primitive generators.

While this work is overall well-structured and designed, it is of comprehensive character and dense in information, which sometimes challenges the reader to follow. In the following I give a list of points where comprehensibility could be improved in order to improve on the accessibility of the work to a somewhat broader audience.

1. I would (this is a matter of taste) avoid stating that things are 'simple','trivial' or 'easy' but refer to being of 'immediate' or 'straightforward' character.

2. On page 2, the mirror symmetry M of the SSH model is exposed, which is a geometrical symmetry of the model. Does chiral symmetry play any role in what comes later ?

3. Page 3, right column, first paragraph. The authors write: "Secondly,....complicated structure." This is a little vague, can you be more precise in your statement.

4. Page 3, right column, second paragraph. The sentence "We then build a lattice...between sites S." is a bit hard to follow: the 'union of sites S' makes it cryptic.

5. Page 3, right column, below equation 11. "The existence of Q is central....of this work." Later on this is not exposed enough. Can you make this clearer why, and later on refer to it again.

6. Page 4, left column, second paragraph. What do you imply by "...a simpler topological characteriation...". The passage followed by "More concretely,..." is somewhat implicity and difficult to follow for the reader, if they had not read the previous latent ssh work. Can you improve on understandability here ?

7. Page 4, right column, first sentence of section "B. Further perspectives...". I think the authors can do better in formulating what they want to express. Try to be balanced.

8. I appreciate that the authors try to put things in a proper context thereby being explanatory. But e.g. in saying "In this case, ... symmetries as well." this seems to be too general to me. The paragraph "There are, nevertheless.....simplifying eigenvalue problem:" could be compactified in one or two sentences. A similar statement holds for the passage "On the other hand....when embedding those unit cells into a lattice." on page 5, left column.

9. Equation 20: Please explain notation (Brillouin zone points of high symmetry, $X_i, Y_i, M...$).

10. The footnote 4 is not understandable.

11. Page 7, right column, second paragraph. The sentence "By carefully analyzing....the latent C_n symmetry." needs more explanation ! What is to be done, how ?

12. Page 9, left column. Statement "...such that the lattice as a whole keeps the symmetry." This is not precise enough. The lattice has a combination of translation plus internal unit-cell symmetry.

13. The sentence "....for clarity, we....is C_n symmetric." is unclear.

14. Page 9, right column. Paragraph below equation 31. Try to explain this a little better, it is clumsy.

15. Concerning figures. The (w,t) phase diagrams need some more explanation in the figure caption.

16. Page 10, right column, below equation 33. If you put w=0 then you have independent SSH chains - that is the well-known standard SSH case. So, why addressing this at all ?

17. Page 12, right column (and several other places), equation 40. I would put the zero blocks as boldface with index, but not as striked out zero.

18. Ref. 21 should be replaced/updated. Page 1, ref.16 citation should be replaced by a more immediately accessible one.

19. The conclusions are not well written, and not well structured in view of the content of the manuscript. I suggest to carefully think over how to do it, and then write it.