## Referee report: Floquet engineering of quantum thermal machines

Dear Editor and Authors,

The author employ an optimal control method for periodic drives to optimize the power output of a periodically driven quantum engine. The manuscript provides a thorough introduction to the field, the specific problem solved, the optimal control method, and the control figures of merit. Without limiting the cycle stroke times, the authors obtain a known result of optimum cycle performance in the limit of vanishing cycle times. To take into account the experimental limitation of band limited pulses, the author introduces an additional term to the control merit function. This leads to optimal control protocols at finite time. I think the paper is interesting, well written and clear. There are however a number of qualitative issues that I think should be addressed before publication.

These points are described below, the two main points are that fluctuations in power are not considered or presented in the analysis. Since, the working medium is a small quantum system fluctuations are expected to play an important, I think it is important to analyze this feature of performance and demonstrate that the engine is indeed useful. If the fluctuation in power greatly surpass the average power, the engine would not be useful. In addition, the authors mention that the Floquet-Lindblidian is the appropriate method to treat the open system dynamics of periodically driven systems. However, this seems to be inconsistent with the choice of Lindblad operators in Eq. (42), as these correspond to the Lindblad operators in the absence of any drive.

## I. PRIMARY COMMENTS

- 1. The authors preformed an optimization of the average power output of continuously driven engine. This is an important figure of merit, however, since the engine is composed of a quantum system fluctuation are expected to play an important role. If the fluctuations are large relative to the average, then the engine should not be really useful. I think it is important to showcase the fluctuations in power or work in order to evaluate the usefulness of optimizing the average power.
- 2. In addition, from various works in the field of quantum thermodynamics related to thermodynamics uncertainty relations. It appears that there should be a tradeoff between optimized power, minimization of fluctuation of power and minimization of entropy production. I think the model studied allows to test and showcase this tradeoff, I think this will greatly benefit the field of quantum thermodynamics and contribute to the novelty of the article.
- 3. In Eq. (42) the Lindblad operators correspond to the Master equation without any driven, and are independent of the drive. This seems to be inconsistent with the statement that the Floquet-Lindblad Master equation is the appropriate way to treat the the open system dynamics of a periodically driven system. In the Floquet-Lindblad Master equation the Lindblad jump operators are transition operators between the Floquet states, which don't correspond to the transition operators in the static case.
- 4. In Ref. [32], there is a discussion of the validity regime of the use of the Floquet-Lindblad (sec. 3.3.1), can the author comment how these considerations relate to the present work.

## **II. MINOR COMMENTS**

1. I think referring to a continuous engine as an Otto engine might be misleading, as the Otto cycle is constructed by four well-defined strokes.