Referee report of "Exact finite-size scaling spectra in quenched and annealed Sherrington Kirkpatrick spin glass"

A. Strengths

- Study of the finite size spectrum close to the edge of a random matrix problem
- Application to the phase transition in the annealed version of the Sherrington Kirkpatrick model
- Comparison with the quenched problem

B. Report

The authors consider the problem of the annealed Sherrington Kirkpatrick model, which was shown to display a subtle transition with the opening of a gap between the bulk and the largest eigenvalue of the exchange matrix below a critical temperature. The aim of the work is to characterise the spectrum of the random matrix representing the coupling between the spins in the large but finite system size. In order to do this they arrive to the analysis of a first order ordinary differential equatio (ODE). Further transformation yields a linear second-order ODE and the nodes of its solution give the exact locations of leading eigenvalues. The study of this equation is amenable of analytical solution in same limit or numerical study. The authors also compute the strength of the different spin components along the eigenvectors of the matrix across the transition and they compare with Monte Carlo simulations.
The article is well written and the analysis is carried out carefully. Some mismatch between the analytical and the numerical results at the critical point is left for future investigation.

C. Requested changes

The authors approximate the problem of Ising spin variables with the spherical model. While the physics in the annealed case is similar the low temperature physics of the quenched problem is different. The authors should probably be careful with this distinction in their discussion?

D. Recommandation

I think that the manuscript meets the criteria of Scipost Physics and I recommend it for publications after that my question is taken into account