Referee Report for "Cosmological Infrared Subtractions & Infrared-Safe Computables"

The manuscript introduces a novel procedure to address infrared divergences in scalar cosmological integrals within perturbative frameworks. By leveraging the combinatorial structures of cosmological polytopes and nestohedra, the authors propose systematic subtraction rules to define infrared-safe computable quantities. These subtraction rules rely on diagrammatic operations on weighted reduced graphs derived from Feynman diagrams. The approach is illustrated with examples, including tree and loop-level computations, to validate the effectiveness of the proposed methodology.

Main Strengths:

- The manuscript offers a systematic, diagrammatic methodology for addressing infrared divergences in scalar cosmological observables. The use of cosmological polytopes and nestohedra to formalize divergence subtraction represents an innovative intersection of combinatorial mathematics and cosmological physics.
- The theoretical foundation is presented systematically. The introduction effectively contextualizes the problem of infrared divergences, grounding the proposed solution in prior work on analytic structures in cosmological observables.
- The derivations and formalism appear thorough, with explicit mathematical details provided for the subtraction rules and their application to logarithmic and power-law divergences.
- The framework has the potential to address divergences in various perturbative settings beyond the specific examples considered in the paper. This adaptability makes it a promising tool for the broader cosmology community.

Weaknesses:

- While the mathematical framework is robust, the physical implications of the subtraction procedure and the resulting infrared-safe quantities are not thoroughly explored. For instance, how these quantities relate to observable features of the early universe remains unclear.
- The manuscript would benefit from a more detailed comparison with alternative methods for handling infrared divergences, such as resummation techniques or holographic renormalization approaches.
- While the procedure is well-defined mathematically, practical implementation details (e.g., computational complexity, numerical stability, and scalability) are not discussed. These aspects are important for the adoption of the method in real-world computations.

The manuscript meets SciPosts criteria for significant, original contributions to theoretical cosmology. The proposed methodology is innovative, and the rigorous mathematical framework aligns well with the journals standards. The suggested improvements below would enhance the papers clarity and broader applicability, but even in its current form, the work represents a valuable contribution to the field.

I recommend the manuscript for publication in SciPost Physics, subject to the authors addressing the points raised above and below.

Additional comments:

- I had to read until page 10 to understand what the authors are actually proposing to do with these “infrared safe computables”. The intro is not clear about what the authors think one should do with these “subtractions”, and what the meaning could possibly be of these ad-hoc constructed infrared computables. I think this is explained in the conclusions in the paragraph “In this paper, ...”. This discussion would be much more useful early on, maybe in the intro.

- Another question that is left unanswered is how many other infrared computable are there and why those defined in this paper are distinct from the infinitely many I could imagine defining. Isn’t is completely arbitrary what I subtract, as long as I remove the IR divergences? What makes one subtraction “better” than another if none of them has a clear physical meaning?

- is it true that all the technical results apply only to a conformally coupled scalar or do the authors have some more general precise result (as opposed to an expectation)? In this regard I was a bit confused by “Importantly, the canonical function (Y,PG) coincides with the flat-space wavefunction contribution associated to the graph G for a conformally-coupled scalar with polynomial interactions”, where a conf. coupled field in Mink. just means a massless field.

- the distinction was to clear to me between general results valid for an arbitrary diagram and specific results for a given diagram. for example, where do eq 17 and 18 come from and why are we looking at those diagrams to begin with? Are those supposed to be the simplest non-trivial examples?

- Similar question goes for the box diagram. Do the author claim to have spelled out a systematic procedure to remove all IR divergences for all diagrams to all loops or just for the box? I did not see/understand the general procedure, if one is presented.

- The derivation of the subtraction procedure is rigorous but dense. Consider including a flowchart or diagram summarizing the main steps of the methodology for clarity.

- “posses” $\rightarrow$“possesses”?