Referee report on "The boundary disorder correlation for the Ising model on a cylinder" by Rafael L. Greenblatt

The article calculates the finite size scaling term of the free energy of the Ising model on a cylinder with periodic and antiperiodic boundary conditions in terms of Jacobi theta functions. This result is then used to provide an explicit expression for the correlation function of disorder insertions for the critical Ising model on a cylinder as a function of the aspect ratio of the cylinder. Finally, the finite size scaling terms calculated in the first part of the paper are checked against conformal field theory predictions and found to agree with them asymptotically at leading order.

The results are interesting and provide a nice addition to the existing literature on finite size scaling for the Ising model. However, the presentation of the results and their derivation can be improved. Indeed, it sometimes feels as if the paper was written in a rush with the intention to keep it short, and with a restricted audience of experts in mind. For the above reasons, I believe the paper can be accepted after a suitable revision. In order to make the paper more accessible and interesting, I think the author should

- expand and clarify the background and motivation,
- explain the notation more clearly,
- briefly summarize the results in the last section (Conclusions).

Below, I provide a more detailed list of comments and questions.

1. Page 2, equations (3-5): The notation is not introduced properly and explained clearly. Referring to paper [1], which is not yet available, does not help the reader. It would help to add a brief discussion of

the Kadanoff-Ceva disorder variables, perhaps with references to some of the original literature (Kadanoff-Ceva, Fradkin-Kadanoff, ...). This could also serve as additional motivation.

2. Page 2, after eq. (6): At this point I started wondering about the ratio $\mathcal{M}/\mathcal{N}$. Commenting on the role of this ratio, especially with reference to Section 4, would not harm.

3. Page 2, after eq. (6): I'm puzzled by the fact that eq. (19) seems to suggest the equivalence $\xi = \frac{2x_1}{1-z_1^2}$, where the right hand side depends only on $z_1 = \tanh \beta E_1$. A comment wouls help.

4. Page 3, line 4: "presented in [15]" ("in" is missing).

5. Page 3, equations (10-14): I guess M, N here are $\mathcal{M}, \mathcal{N}$, respectively.

6. Bottom of page 3: I don't find the notation very clear, it would help to state explicitly how the blocks are arranged to form the matrix $\mathbf{A}$.

7. Page 4, line 1: I assume the last "case" refers to the last block/matrix.

8. Page 4, line 1: The parity of what?

9. Page 4, between eq. (19) and eq. (20): The new line should start with a capital letter.

10. Page 4, between eq. (19) and eq. (20): Why are α_1 and α_2 introduced here? They don't seem to be used anywhere.

11. Page 4, line before eq. (23): I guess "the limit" here refers to $\mathcal{M}, \mathcal{N} \to \infty$ with $\mathcal{M}/\mathcal{N}^2 \to 0$. It wouldn't harm to mention this explicitly.

12. Page 5: The derivation of the coefficients in (32) is not clear to me.

13. Page 6, line after eq. (33): Why is it sufficient to consider the case $\theta \to 0$? What about the contribution of other values of θ in $Q_{\pm}(N)$?

14. Page 7, line after eq. (44): I believe "the other expressions obtained above" means equations (43), (31), (32) and (1). It would help the reader to make this explicit.

15. Conclusion, line 5: When considering $\mathcal{M} >> \mathcal{N}$, it may be worth mentioning again that $\mathcal{M}/\mathcal{N}^2 \to 0$.

16. Page 7, equations (48), (52) and (53): I think z_+ and z_- should be x_+ and x_-.