Report for the paper: *The boundary disorder correlation for the Ising model on a cylinder*

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The paper presents new results about the correlation function of *disorder insertions* for a cylindrical Ising model at criticality with open and periodic (anti-periodic) boundary conditions. Following the theory of Kadanoff-Ceva for disorder operators, the correlation can be written indeed in terms of the ratio of the partition function with defects and without. In particular, the ratio considered is between the partition function with open/periodic boundary condition and the partition function with open/antiperiodic boundary conditions. Finite size corrections to the pressure are provided and shown to be consistent with CFT's predictions.

Despite the results are new, I found the article difficult to read and in several places imprecise. Starting from the introduction, it seems confused and without a solid structure. The paper starts with "I study the disorder correlation" without even explaining the notations of formulas (1) and (2). The author should put this results in a more general context. For instance, there should be a clearer link to Kadanoff-Ceva theory and the fermionic interpretation. Moreover,in the introduction formulas are given without explaining the variables. All this makes the article very hard to read.

Furthermore, I have the perplexities about the novelty of the paper. The core of the article is the asymptotics of the partition function using the McCoy-Wu solution for the Ising model in cylindrical boundary conditions. The author already reviewed this method (in a simplified form) in the (clearer) paper *The Ising model on a cylinder: universal finite size corrections and diagonalized action*, where the exposition is more formal and less sketchy. The results of Sections 2,3 are straightforwards generalizations to the anisotropic case and to the different boundary conditions of Sections 3 and 4 of the mentioned paper.

Hence, I would not recommend the publication of the submitted article. However, since the manuscript I have mentioned (i.e., *The Ising model on a cylinder: universal finite size corrections and diagonalized action*) seems to appear only in the arxiv, a well written review of the finite scaling of the Ising model on a cylinder with general boundary conditions (including these new results) might deserve to be published.

I have the following minor remarks:

1. Explanations about the quantities in formulas (1)-(6) are due.

2. Explain the different notations for the θ functions: $\theta_k(\cdot,\cdot)$ and $\theta_k(\cdot|\cdot)$ in formula (6).

3. In formula (7) I would emphasize that p, s are constants that do not depend on the boundary conditions.

4. There is a different notation used in formula (2) and in formulas (10)-(15): M, N instead of $\mathcal{M}, \mathcal{N}$.

5. I think formula (10) is wrong if the partition function is as in (2): I expect a $4MN$ in the exponent instead of MN and $2N(M-1$ instead of $N(M-1)$.

6. The quantity t in the matrices (12) and (14) it is not introduced: is it the critical temperature of the system or z_i?

7. In the text after formula (15) there should be *product* instead of *sum*.

8. The description for the matrices $B(\theta)$ is too vague. If they are really the same as of the ones in equations (3.6a) and (3.6b) of the reference [23], why the notation has an explicit dependence on the boundary conditions?

9. The sentence after equation (44) is also too vague: which other expressions? Please mention them. Moreover, please show how you get equation (6) in more details.

10. I would not call Section 4 **Conclusions** but rather something like **Comparisons with CFT's predictions**.

11. I would also recall results for the predictions for general cylindrical symmetry, e.g., $\pi/24$ for open boundary conditions.