Referee report on *Boundary SymTFT*
(scipost.org/submissions/2409.02166v2)

This paper discusses aspects of boundary conditions, as well as interfaces (or defects), in quantum field theories with noninvertible symmetries from the perspective of the SymTFT paradigm. To this end an additional gapped boundary is introduced to the SymTFT, resulting in what the authors refer to as a "Boundary SymTFT".

This approach is definitely new and interesting. The basic picture, illustrated in Figures 1 and 2, is intuitively appealing. Also, concrete realizations of the approach are exhibited in detail in various examples in dimension 2 and 3. On the other hand, despite struggling for a considerable time I have not been able to grasp the meaning of some of the specific arguments. Still, altogether I think that the paper is worth being published.

There is, however, one aspect in which in my opinion the paper must be revised before it can be published. This concerns the higher-categorical tools that are invoked, specifically fusion $(d-1)$-categories, module categories over them and the dual fusion category of module endofunctors, as well as the notion of Drinfeld center.

Fusion 2-categories have already been introduced in 1812.11933 (still unpublished), and module 2-categories over those in 2107.11037 (also unpublished), and important aspects of their theory are now well studied. Also general (weak) n-categories have been modeled in various ways. In contrast, to the best of my knowledge, for $n \geq 3$ no general theory of fusion n-categories or module n-categories that would e.g. allow to discuss aspects of strictness (which become more intricate when $n \geq 3$) and pivotality/sphericality is fully available. (As far as I understand, the fusion n-categories considered in the literature, such as in 2408.13302, form a special subclass that can be understood in terms of an iterative procedure.)

I suggest that that the authors state this situation clearly in the introduction and provide some proper references.

I also have the following specific comments:

• At instances like the list (2.11), "module categories" should be "indecomposable module categories".

• The internal-Hom construction invoked after (2.32) applies to $d = 2$. Is it claimed that there is an analogous construction also for higher d?

• Why is the category $\mathcal{N}$ appearing in (2.33) a module category over $\mathcal{S}^\star_{\mathcal{M}}$?

• Instead of $\varphi^S_{m,n}$ in (3.2) being, as stated, in $Hom(S \otimes m, n)$ (which I interpret as $Hom_{\mathcal{S}}(S \otimes m, n)$), I would have expected it to be in $Hom_{\mathcal{M}}(S \otimes m, n)$.

• In which sense is (3.169) a *map*?

Further, I noticed that there is a sizeable number of miswritings and of typos, including such which could easily be detected by a spell check. Some examples of the former are: "defect lines of Verlinde lines", "theory is gapped phase", "for

which have", "In this way has two". Among the typos are "$\in\in$" (before (2.27)), "0-from", "pratice", "lagebras", "bot sides". And in the caption of Figure 2, "Q_2" should presumably be "Q_d" (twice).

As a final observation, it seems that many of the references given in the first two paragraphs of the Introduction (concretely, [1, 2, 4–23, 26–28, 30, 32, 33, 35–37, 40]) do not appear elsewhere in the text. Are all of them needed for understanding the contents of the paper?