Referee's Report on the paper "Massless chiral fields in six dimensions"
by Thomas Basile

The paper investigates massless chiral fields of arbitrary spin in six-dimensional spacetime, referred to as higher-spin singletons, and presents a framework for describing these fields using symmetric $SL(2,\mathbb{H})$ tensors. The author aims to extend concepts from four-dimensional higher-spin theory, where the fields are described using a combination of 0-forms and 1-forms that take values in $SL(2,\mathbb{C})$ tensors. The paper also explores interactions involving these massless fields and extends the formalism to arbitrary even-dimensional spacetimes.

In my opinion, the paper makes an important contribution to the field of higher-spin theories by providing a novel framework for describing massless chiral fields in six dimensions. The results offer promising avenues for further research, particularly in the development of interacting theories and their extension to higher dimensions. The presentation is well-organized, and the connections to related mathematical structures are compelling. Therefore, I am pleased to recommend the paper for publication in SciPost Physics.

Below are some comments and suggestions for improvement, most of which are optional.

1. The sentence "Such massless fields are known to be conformal, ..." before Eq. (1.21). It is not quite clear which "such massless fields" are referred to. What is the precise relation between the off-shell strength F and the fields Ψ and Φ subject to Eqs. (1.21) and (1.22)?

2. In Eqs. (1.21, 1.22) and below. What is the significance of using the wavy equality sign instead of the standard one? Not all readers may be familiar with the theory of constrained Hamiltonian systems, from which this symbol is apparently borrowed.

3. At the beginning of the subsection "Reducibility of the gauge symmetries," the author mentions "the correct number of degrees of freedom" for the free equations (1.21, 1.22), but provides no indication of what it might be. It seems that the author's argument here is that the (gauge equivalence classes) of on-shell fields carry a unitary irreducible representation of the conformal group $SO(2,5)$, which is a characteristic property of singletons. If so, it would be nice to state this explicitly.

   At the same time, there is a more direct way to convince oneself that equations (1.21, 1.22) do propagate the correct number of physical degrees of freedom, without appealing to the conformal group. To this end, one can use a general formula for the physical degrees of freedom from Ref. [1, Eq.(80)]. When applied to Eq. (1.22), it reduces to

   $$N = 1 \cdot t - 1 \cdot r + 2 \cdot r_1 \,,$$

   and r_1 is the number of redundant gauge symmetries. Using the well-known formulas for the dimension of the space of totally symmetric tensors and tensors with hook symmetry, we readily find

   $$t = \frac{(p+1)(p+2)(p+3)}{6} \,, \qquad r = \frac{(p^2-1)(p+2)}{2} \,, \qquad r_1 = \frac{(p^2-1)p}{6} \,,$$

where $p = 2s$. Combining these numbers, we get $N = 2(2s+1)$. This is the number of physical degrees of freedom per point in the phase space of fields. The number of physical polarizations of the field Φ is thus equal to $2s + 1$. It remains to note that $2s + 1$ is exactly the dimension of the irrep of the little group $SU(2) \times SU(2)$ whenever either of the two factors acts trivially. A similar analysis can be repeated for the field Ψ. Although the field equations (1.21) have no gauge freedom, they enjoy (reducible) gauge identities, which can also be taken into account with the help of the general formula. Actually, as the system(1.21) and (1.22) are dual to each other, they are known to describe the same number of physical degrees of freedom. A rigorous proof of this fact can be found in the recent paper [2].

4. Line after (1.42): It might be better to replace "the two-raw Young diagram of so(1,5), ..." with "the two-raw Young subdiagram of so(1,5)".

5. Section 1.2 dresses the coupling to a Yang-Mills field. I think the term "Yang-Mills field" for the field A is somewhat misleading in this context. Usually, Yang-Mills fields refers to both the fields and their equations of motion. The fields A and B define a BF-modal and carry no physical degrees of freedom, unlike YM theory. It might be better clearer to refer to A as a "gauge vector field" or a "connection 1-form".

6. Line before (1.55): "this bilinear form verifies" $\longrightarrow$ "this bilinear form satisfies".

7. There is no coupling constant g in the definition of the covariant derivative (1.56). However, this constant pops up in Eq. (1.60). Clearly, it can be absorbed by the re-definition of the field B.

8. The sentence around (160) and (161) is too wordy and unclear. Please consider splitting it into two or three sentences for better clarity.

9. The comment before Eq. (1.62) is somewhat confusing. Equations of motion know everything about their gauge symmetries, reducibility, and physical degrees of freedom. Therefore, any choice of gauge symmetry generators (1.59) – with or without F-term – would lead to reducible gauge transformations and correct number of physical degrees of freedom. Since $F \sim \delta S/\delta B$, the F-term vanishes on-shell, representing thus a natural arbitrariness in the choice of gauge symmetry generators.

10. The sentence around (1.73) and (1.74) is too wordy and unclear. Please, split it into two or more sentences.

11. Paragraph after (1.76): "and that the Lie bracket is understood..." $\longrightarrow$ "and the Lie bracket is understood ...".

12. It is worth stressing that the cubic term (1.85) is a peculiar property of chiral theory in six dimensions.

13. In the context of higher-spin gravity, the presymplectic AKSZ action (1.95) was introduced and studied in Ref. [3]. Among other things, in that paper, the first order correction to the free presimplectic potential (Ψ^2-term) was explicitly constructed, see Eq. (6.43) and Footnote 23.

References

1. Kaparulin, Dmitry S., Simon L. Lyakhovich, and Alexey A. Sharapov. "Consistent interactions and involution." Journal of High Energy Physics 2013.1 (2013): 1-31; arXiv preprint arXiv:1210.6821 [hep-th].
2. Lyakhovich, Simon, and Dmitri Piontkovski. "Degree of freedom count in linear gauge invariant PDE systems." arXiv preprint arXiv:2501.16042 (2025).
3. Sharapov, Alexey, and Evgeny Skvortsov. "Higher spin gravities and presymplectic AKSZ models." Nuclear Physics B 972 (2021): 115551; arXiv preprint arXiv:2102.02253 [hep-th].