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In this work, the authors study the effect of the uniform magnetic field on the equilibrium properties and non-equilibrium dynamics in the three-state Potts spin chain model in the ferromagnetically ordered phase. This model is the direct generalisation of the quantum Ising spin-chain model to the case of three allowed spin directions. There are a lot of similarities in properties of these two models. In the ferromagnetic phase at zero magnetic field, the ground states in both models are q-fold degenerate, with $q = 2$ and $q = 3$, in the Ising and the three-state Potts cases, respectively. In both models, application of an arbitrary weak uniform magnetic field leads to a dramatic impact on their properties. Such a field removes the degeneracy between different ferromagnetic vacua, and induces confinement and/or the Wannier-Stark localisation of kink topological excitations, as well as the false vacuum decay. In the case of the quantum Ising spin chain, all these phenomena are well established and have been thoroughly studied in the literature both analytically and numerically. The goal of the authors of the present manuscript was to extend the analysis of these phenomena to the less studied, but more rich and difficult case of the three-state Potts spin chain. Its difficulty stems from the fact, that at generic values of the coupling parameter g, this model is not integrable already in the absence of the magnetic field, in contrast to the Ising spin chain model. Because of lack of integrability of the three-state Potts spin chain both in confinement and deconfined regimes, the authors of this work mostly concentrate on the numerical study of its finite-size version. To this end, they apply several techniques, which were previously proved to be effective in the case of the Ising spin chain.

The authors investigate the long-lived oscillations in the time evolution of observables arising after the global quantum quenches in the three-state Potts spin chain. Four quench protocols have been invoked, in which the perturbing magnetic field was applied in four different directions. In the cases of the magnetic field parallel and anti-parallel to the initial magnetisation, the observed post-quench dynamics was to much extent analogous to the Ising case. In particular, the long-lived oscillations of the system magnetization were indicated, and their frequencies were found to be close to the energies of two-kink kink bound states ("mesons", or "bubbles"), which were determined in independent numerical calculations. Besides, the suppression of the spreading of correlations after the quantum quenches was also observed by the authors. In analogy with the previously studied quantum quenches in the Ising and XXZ spin chains, this suppression is explained as the result of to the kink confinement and/or their Wannier-Stark localisation induced by the magnetic field.

Aside from this, the authors reported several new features, which are specific to the ferromagnetic three-state Potts spin chain, and are absent in the more simple Ising spin-chain model. In particular, they observed the contribution of baryonic (three-kink) excitations in the post-quench oscillation of the magnetization. Two oblique quench regimes studied in the manuscript, in which the longitudinal magnetic field is not parallel to the initial spontaneous magnetization, also have no analogues in the Ising model.

The result described in this manuscript are new, valuable, and interesting both for theoretical and experimental researchers. The paper is well written and accessible. I recommend its publication in SciPost Physics with minor revision in response to my following two comments.

The first one relates to the presentation of the results of numerical simulations in Figures 5.1 - 5.4. I think, it would be useful to indicate by vertical gridlines the lower $\tilde{\omega}_{min}$ and upper $\tilde{\omega}_{max}$ bounds of the two-kink continuum spectrum in the pure transverse chain (A.1) in the thermodynamic limit $L \to \infty$. This would help the readers to distinguish collisional and collisionless mesons/bubbles, since their energies lie inside, and outside the interval $(\tilde{\omega}_{min}, \tilde{\omega}_{max})$, respectively. According to equations (A.3), and (A.4), these bounds are located at $\tilde{\omega}_{min} = 2$, and $\tilde{\omega}_{max} = 2\sqrt{A-B}/\sqrt{A+B} \approx 2.66$.

The second comment relates to the author's interpretation of their computer simulation results in the positive and negative oblique regimes. In page 27 of the manuscript, in which the oblique quench with $h_2 > 0$ is discussed, the authors state:

> Unfortunately, the effective Ising kink-antikink states mix with the bubble spectrum, complicating their identification in the ED results.

A similar statement is given in page 29, where the oblique quench with $h_2 < 0$ is described:

> As in the previous case, disentangling these two-kink states from the meson states is not always possible, and the orange dashed lines denote the states for which it could not be achieved.

To my opinion, it is in principal impossible to distinguish the stable two-kink states from the collisional meson/bubble states in both positive and negative oblique regimes $h_2 \neq 0$. The reasons supporting this claim are the following.

Since the matrix element $s_2(k_1, k_2)$ of the S-matrix given by equation (A.10) is non-zero, the two-kink in-state $|K_{12}(k_1)K_{21}(k_2)\rangle$ in the infinite chain at $h_2 = 0$ can easily transform after the kink scattering into the out-state $|K_{13}(k_2)K_{31}(k_1)\rangle$. Therefore, in both oblique regimes at $h_2 \neq 0$, the collisional mesons/bubbles cannot exist in the infinite chain as stable excitations of the first vacuum: such mesons/bubbles would easily decay into the unlocalised state of two stable kinks $K_{13}K_{31}$. These two kinks move away one from another, do not interact at large distances, and never meet again. However, these two-kink states $K_{13}K_{31}$ should be hybridised at $h_2 \neq 0$ with the collisional mesons/bubbles. In the case of infinite chain, such hybridised states should form a continuous energy band. The energy levels in the finite spin chain, that fall inside this band, correspond to such hybridised states. This means, that all dashed lines in Figures 5.3, and 5.4 with energies $\tilde{\omega} \in (\tilde{\omega}_{min}, \tilde{\omega}_{max})$ should be rather shown in orange, since they represent the two-kink $K_{13}K_{31}$-states hybridised with the collisional meson/bubble states. On the other hand, the collisional mesons/bubbles can still exist in the oblique regimes $h_2 \neq 0$ of the infinite chain as unstable particle (resonances). In the Fourier spectra of the post-quench oscillations in the finite spin chain, these unstable (collisional) mesons/bubbles should manifest themselves as wide resonant peaks. It is likely, that the authors indeed observed two such resonant peaks in $|M_1(\tilde{\omega})|^2$ in the oblique regimes at $h_2 = \pm 0.1$, and one resonant peak at $h_2 = \pm 0.2$. These wide peaks are clearly seen in the left subfugures of Figures 5.3, and 5.4. If this interpretation is correct, the widths of these resonant peaks should approach some non-zero values ("natural resonance widths") upon increase of the spin-chain length.

I would like also to draw attention of the authors to the recent papers listed below.

1. J. H. Robertson, R. Senese, and F. H. L. Essler, *Decay of long-lived oscillations after quantum quenches in gapped interacting quantum systems*, Phys. Rev. A **109**, 032208 (2024).

2. S. Darbha *et al.*, *Long-lived oscillations of metastable states in neutral atom systems*, Phys. Rev. B **110**, 155114 (2024).

3. G. Lagnese, F. M. Surace, S. Morampudi, and F. Wilczek, *Detecting a long-lived false vacuum with quantum quenches*, Phys. Rev. Lett. **133**, 240402 (2024).

4. S. B. Rutkevich, *Spinon confinement in the gapped antiferromagnetic XXZ spin-1/2 chain*, Phys. Rev. B **106**, 134405 (2022).

Note, that Section IV.B of the last paper contains the detailed description of the heuristic semiclassical calculation of the meson energy spectra in the XXZ spin-chain model in the dynamical confinement Regimes I, II, and III, which are also discussed in the present manuscript.

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