REFEREE REPORT ON "BV FORMALISM AND PARTITION FUNCTIONS"

The authors study the counts of degrees of freedom in free classical field theories and generating functions for them – Hilbert-Poincaré series (for the fiber of the jet bundle of fields), which are called "partition functions" in the paper. This is done in a large collection of examples (including free gauge theories of 1- and 2forms, their self-dual and supersymmetric versions, gravity). The counts of degrees of freedom are done using Batalin-Vilkovisky (BV) formalism, where instead of taking into account the effect of gauge symmetry, equations of motions and their syzygies on the degrees of freedom one has to account for the (higher) ghost fields and anti-fields present in the BV complex.

Comments.

- (1) The paper is in dire need of proofreading. There are all sorts of English grammar mistakes wrong prepositions, wrong articles, mistakes with singular/plural, wrong word order in sentences. Also there are plenty of typos. This makes it unnecessarily hard to read the text, and it sometimes obscures the mathematical message. (E.g., already in the abstract, I suspect that instead of "partition functions of gauge invariant operators" it should be "partition functions and [spaces of] gauge invariant operators," which changes the meaning.) This all leaves the impression that the paper was written sloppily, in haste. Maybe it would make sense to ask someone who is a native English speaker to help proofread the text.
- (2) Sometimes there are strange choices of words, like "engineering dimension" (middle of p.4). In footnote 1, "weights" of a fugacity t, should probably be instead "powers" of a fugacity t.
- (3) In Section 3, as a suggestion, it might help the exposition to start with the example of the free scalar field before going to 1-form gauge fields.
- (4) Typo in formula (3.9), p.7: the denominator in the first term should be $(1-t)^4$ instead of $(1-t^2)^4$.
- (5) Typo in formula (3.23), p. 11: in the brackets in the r.h.s., it should be $(1+t^2)$, not $(1-t^2)$.

Conclusion. I found the paper to be interesting and thought-provoking. Especially, I found the connections to world-line formalism in Section 3.1 and a putative generalization in Section 6 to be fascinating.

As an aside, it is curious that some BV complexes used in the paper (notably, for self-dual gauge fields, and also the complex in the bottom third of p.6 for the Maxwell field, using the extra fields ϕ, ϕ^*) are not Poincaré complexes – they do not come with a compatible Poincaré duality pairing. That means that such a complex is not generated by a master action via a Poisson structure associated to a degree -1 symplectic form. However, there could be a quasi-isomorphic form of the same BV complex which does support Poincaré duality and is generated by a master action.

I am happy to recommend the paper for publication in SciPost Physics, after a minor revision: I'd suggest my comments above to be taken into account – especially, the comment (1) – proofreading.