1. On page 2: Although the authors say that the matrix A is identified with the mixed Chern-Simons level matrix K as describe in eq.(1.5), it will cause misunderstanding as the matrix A that appears in the Dirichlet half-index is generally determined by "boundary data" (as well as the bulk theory data) since it encodes the boundary 't Hooft anomaly for a specific choice of boundary conditions. In other words, for the theory described as eq.(1.5), A generally depends on r due to the boundary conditions satisfied by the chirals. Why is it independent of r? Also it is rather strange why they do not mention about the matrix B which is also determined by the boundary conditions.

2. On page 10: They should improve explanation about the half-index and the boundary conditions around eq.(3.27) in addition to the previous point raised. Following the prescription in the paper [54], the Dirichlet half-index can be computed from the boundary 't Hooft anomaly that depends on a choice of the boundary conditions. The determination of the form (3.27) is less obvious as such discussion is missing. Why is it fixed only by the bulk data K, a and μ introduced in section 3.1 without any information about the boundary conditions?

3. On page 10: Although they briefly say the Dirichlet boundary condition around eq.(3.27), precisely speaking, this should be stated as the Dirichlet boundary condition preserving $\mathcal{N} = (0, 2)$ supersymmetry, while there also exists the Dirichlet boundary condition preserving $\mathcal{N} = (1, 1)$ supersymmetry. It would be better to describe the boundary conditions they are choosing more precisely.

4. On page 11: In eq.(3.27) the specialization $\nu = 1$ is identified with the topological A-twist. Similarly, they propose that the B-twisted Wilson line is related to the A-twisted one according to eq.(3.28). As discussed in section 1 the twisting can be performed if the theory has enhanced supersymmetry. But I am skeptical about this interpretation. Such enhancement may not occur in the case with boundary. If some of the boundary conditions only preserve $\mathcal{N} = (0, 2)$ supersymmetry albeit the enhanced $\mathcal{N} \geq 4$ supersymmetry in the bulk, their physical interpretations as the topological A-twist and B-twist will be inappropriate. Instead, such a specialization may be rather simply interpreted as an insertion of the flavor Wilson line since it only results in a shift of the power of q. This can be non-trivial even though the boundary conditions only preserve $\mathcal{N} = (0, 2)$ supersymmetry. In this regard, I am not sure whether the physical interpretation of the specialization of the fugacity as well as the notation I_A are reasonable.

5. On page 12: This issue is related to the item 1. In eq.(1,5) they state that

they study 3d $\mathcal{N}=2$ $U(1)_K^r$ Chern-Simons theory with r chirals, however K in section 4 does not seem to match the CS level of the bulk theory. For example, in section 4.1.1 the theory with $r=1$ and $K=2$ is identified with the one found in [57], that is the $U(1)_{-3/2}$ gauge theory of CS level $-3/2$. I am very confused with the convention.

6. On page 12: The identification of the Dirichlet half-index of the $U(1)$ gauge theory of CS level $3/2$ with the character of the Virasoro minimal model $M(2,5)$ already appeared in the paper of Okazaki-Smith, "Line defect half-indices of $SU(N)$ Chern-Simons theories", JHEP 06 (2024) 006, arXiv:2403.03439. In eq.(3.17) of the above paper $\chi_{1,2}$ is identified with the Dirichlet half-index without any insertion of the line, whereas it is realized as the Dirichlet half-index with a Wilson line in section 4.1.1. The difference will be simply because the authors are considering the specialization with $\nu=-1$ in (3.27), which corresponds to an insertion of the flavor Wilson line.

There are a few minor typos.

1. On page 10: "cancel in cancel in" will be a typo.

2. On page 11: "'The basic mirror duality is expressed as[40, 55]". There will be a space after "as".