Report for SciPost-2411.18685v3
- All 4×4 solutions of the quantum Yang-Baxter equation -

The Authors address the important problem of 'classifying' 4×4 solutions of the quantum Yang-Baxter equation. The latter finds applications in both Theoretical Physics and Mathematics and more recently in quantum computing and hence it is an interesting and essential problem to be considered. They use a method that was previously developed by one of them. The paper is definitely of interest and deserves to be published, however it lacks certain crucial details.

1. The assumptions under which the Authors call this scheme a 'classification' has to be clearly laid out.

2. We found several non-regular 4×4 solutions that are missing in the Authors' list of solutions. These are:

   (a) $\begin{pmatrix} 1 & 0 & 0 & f(u,v) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ - This solution is inequivalent to Eq. 26 on page 5. The equivalence relations are defined by the continuous and discrete symmetries of the Yang-Baxter equation as found in [1].

   (b) $\begin{pmatrix} 1 & f_1(u,v) & f_2(u,v) & f_3(u,v) \\ 0 & 1 & 0 & f_2(u,v) \\ 0 & 0 & 1 & f_1(u,v) \\ 0 & 0 & 0 & 1 \end{pmatrix}$ - Here f_1, f_2 and f_3 are arbitrary functions in the spectral parameters u, v. It satisfies the non-additive quantum Yang-Baxter equation. This solution can be seen as a deformation of $(2,3)$ equivalence class of Hietarinta [1]. It is inequivalent to all the solutions listed by Authors under the continuous and discrete transformations of the YBE.

   (c) $\begin{pmatrix} 1 & a & -a & cu+ab \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & -b \\ 0 & 0 & 0 & 1 \end{pmatrix}$ - Here a, b and c are complex constants. The spectral parameter is u. This solutions satisfies the additive quantum Yang-Baxter equation. It is a

deformation of the $(1,3)$ equivalence class of Hietarinta [1]. This solution is also inequivalent to all those listed by the Authors.

3. There are also other interesting solutions where the spectral parameters are three dimensional. See Page 126 of [2]. These are non-regular and 4×4 solutions. The solution is based on the group $SL(2,\mathbb{C})$.

We have only analyzed the invertible Hietarinta solutions to find the above listed matrices that are non-regular and inequivalent to those listed by the Authors. We can carry out a similar analysis for the rank 3, 2, and 1 solutions to find more inequivalent 4×4 solutions.

Given this I think the title of the paper "All 4×4 solutions.." is rather misleading and thus needs modification. I suggest the paper be sent back to the Authors for major revisions.

References

[1] Jarmo Hietarinta. All solutions to the constant quantum yang-baxter equation in two dimensions. *Physics Letters A*, 165(3):245–251, 1992.

[2] Vladimir E. Korepin, Nikolai M. Bogoliubov, and Anatoli G. Izergin. Quantum inverse scattering method and correlation functions. Cambridge university press, 1993.