Report on: Grey Galaxies in AdS_5

Rotating AdS_d black holes (Kerr-AdS_4 or their higher-dimensional partners) with an angular velocity that exceeds the speed of light, $\Omega_H \ell > 1$ (ℓ is the AdS radius) are known to be unstable to superradiant perturbations [2, 3, 4, 5, 6]. In a phase diagram of stationary solutions (for example $\Omega_H \ell$ vs the areal radius r_+/ℓ or dimensionless energy E vs angular momentum J), the onset of the m-mode instability signals a bifurcation to a new family of solutions that describe m-mode black resonators [7, 8, 9, 10, 11]. For a given energy and angular momentum, black resonators have higher entropy than the original Kerr-AdS black hole (when they co-exist) and they exist all the way till the BPS curve, $E = J$. Thus, Kerr-AdS black holes are not the unique rotating solutions of Einstein-AdS theory. These black resonators are not time independent neither axisymmetric but they are periodic since the vector field that generates the horizon is a Killing vector field of the solution. Interestingly, m-mode black resonators are still superradiant unstable since they still have $\Omega_H \ell > 1$ [4, 5, 16, 15] (in particular, there is no black resonator with $\Omega_H \ell = 1$ [12]). More precisely, they are unstable to $\tilde{m} > m$ perturbations. The entropy of m-mode black resonators increases with m (for a given energy and angular momentum). This suggests, as conjectured in [7, 5, 12], that the time evolution of the superradiant instability of Kerr-AdS black holes should proceed via a cascade, with increasingly higher m, of metastable configurations that resemble temporarily m-mode black resonators until a violation of weak cosmic censorship occurs. In the sense that, at very large m, the Planck scale is reached and thus Einstein-AdS gravity can no longer describe the evolution of the system and quantum gravity is required. This conjecture, at least the initial/intermediate stages of it, has been validated by available numerical time evolution studies [13, 14]. However, a big question remained unanswered: what could be the endpoint of the superradiant instability of Kerr-AdS? It cannot be a black resonator, no matter how arbitrarily large m is, because such a black hole still has $\Omega_H \ell > 1$. Ref. [1] proposed that the endstate should be a Grey Galaxy and this solution was explicitly constructed. This is a system made of a central Kerr-AdS black hole with $\Omega_H \ell = 1$ and a disk of rapidly rotating gravitons (or a gas of scalar field or photons if the original system starts with bosonic perturbations with spin 0 or 1) at very large radius and sharply localized along the equatorial plane.

The present manuscript extends non-trivially the analysis of Ref. [1] to AdS_d with an emphasis on $d = 5$ (but with many aspects of the analysis also holding for higher dimensions). The analysis includes AdS_d backgrounds but also supergravity backgrounds with internal manifolds (e.g. $\mathrm{AdS}_5 \times S^5$). In 5-dimensions, one has two rotation planes and thus Kerr-AdS_5 black holes can have angular velocities $\Omega_H^{(1)}$ and $\Omega_H^{(2)}$ along these two rotation planes. This opens the door for the existence of novel black resonator configurations and thermodynamic competitions between them. It also introduces novel Grey Galaxy configurations which are the main object of study of this paper. Namely, the present manuscript identifies two qualitatively distinct Grey Galaxy configurations: the first class has either $\Omega_H^{(1)} \ell \approx 1$ and $\Omega_H^{(2)} \ell < 1$, or $\Omega_H^{(2)} \ell \approx 1$ and $\Omega_H^{(1)} \ell < 1$. The second class is a Grey Galaxy with both angular velocities parametrically equal to one: $\Omega_H^{(1)} \ell \approx \Omega_H^{(2)} \ell \approx 1$. The former Grey Galaxies are qualitatively similar to AdS_4 Grey Galaxies but the latter Grey Galaxies have several unique new properties. For example, the gas of rotating gravitons is smoothly distributed over the full spatial bulk and the holographic stress tensor is smoothly distributed over the whole boundary surface, instead of being localized over its equator. Interestingly, the holographic stress tensor of the gas component of the second class of grey galaxies is shown to have a conformal fluid form for any temperature (not only high), and the reasons that might explain this unexpected property are identified. The $\Omega_H^{(1)} \ell \approx \Omega_H^{(2)} \ell \approx 1$ Grey Galaxy is explicitly constructed − and this is a remarkable nontrivial task − when the bulk matter is a massless scalar field in $\mathrm{AdS}_5 \times S^5$, but the authors use this example to produce an educated guess for the boundary stress tensor for arbitrary bulk matter. The authors display all the grey galaxies in a phase diagram of angular momenta versus energy and identify the regions where each solution exists as well as regions where each of them dominates the microcanonical ensemble. Although the manuscript addresses objectively several questions, it also raises several other questions/conjectures that would be interesting to address in future studies.

The manuscript is very well written and has unequivocal high level of novelty. It should also be of unquestionable interest for the gravitational and string theory communities. Globally, this is a beautiful and ingenious scientific work. It also contains interesting open questions and conjectures that open a new pathway in an existing research direction and have clear potential for multi-pronged follow-up research. Thus, I strongly recommend its publication in SciPost Physics (below I just provide some comments that might contribute to enhance two or three discussions).

There are a few discussions in the paper about the conjectured complete phase diagram of solutions (and thermodynamic competitions between them) that might benefit from the following observations:

1) Starting in the first paragraph of page 4 (associated with footnote 12; see also section 3 and page 44), the authors discuss how $\mathrm{AdS}_5 \times S^5$ black holes localized on the S^5 (that approach 10-dimensional Schwarzschild or Kerr in the small energy limit) might compete with the $\mathrm{AdS}_5 \times S^5$ Grey Galaxies. It should be noted that the localized $\mathrm{AdS}_5 \times S^5$ black hole family that the authors refer to has been explicitly constructed in [17] (when the black hole is not rotating) and the associated thermodynamic properties have been computed, including the identification of the critical point where the phase transition between the uniform (on S^5) and localized phase (on S^5) occurs. Although the system in [17] has no rotation in AdS_5, maybe the data there could be used to formulate an educated guess about what happens when rotation is added. This might shed light on the competition between rotating localized $\mathrm{AdS}_5 \times S^5$ black holes and $\mathrm{AdS}_5 \times S^5$ grey galaxies. The authors seem to be unaware of this study since they cite the companion paper [18] (that studies the lumpy or clumpy black holes along the S^5) but not [17] that found the localized black holes on S^5 and their properties.

2) In the first paragraph of section 1.6 (page 7) it is stated that "*The Gregory-Laflamme instability always occurs at values of ω (i.e. $\Omega_H \ell$) that are greater than unity, and so for black holes that were already super radiant unstable*"
(A similar statement can be found in Appendix A). Is this statement correct? At first sight, I would say that what matters for the Gregory-Laflamme instability is the size of the horizon radius r_+ compared with the S^5 radius ℓ. Thus I would expect that black holes with small r_+/ℓ would be Gregory-Laflamme unstable no matter what their angular velocity is (rotation tends to enhance the Gregory-Laflamme instability [19]). In particular, they should be Gregory-Laflamme unstable even for small rotation where the system is not superradiant unstable. And if I understand them correctly, the analyses of [6, 19] show this is indeed the case.

3) In AdS_d with $d \geq 6$ (but not $d = 5$), besides Kerr-AdS, AdS black rings, black resonators and Grey Galaxies, there is another class of black holes that appear in the phase diagram of solutions. These are the "(ultraspinning) lumpy black holes" that bifurcate from the onset of the ultraspinning instability of Kerr-AdS_d [20]. These lumpy black holes

– which are axisymmetric rotating black holes with lumpy or rippled S^{d-2} horizons (i.e. the S^{d-2} horizons are deformed along the polar direction of the sphere S^{d-2} of AdS_d) – were found in [21] (asymptotically flat case) and in section IX of [22] (asymptotically AdS case). They connect Kerr-AdS_d black holes with AdS_d black rings in a phase diagram of solutions (when $d \geq 6$) so they should also contribute to the discussion in Appendix A.

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