Referee Report

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In this manuscript, the authors propose a realization of 2d symmetry-preserving quantum RG (SymQRG) flow from a 3d quantum gravitational path integral. The manuscript is written in the self-contained way. The construction of this framework is demonstrated concisely step by step.

The authors start with an systematical explanation of difference between conventional Wilson-Kadanoff RG and SymQRG. Given the $T\bar{T}$ deformation of 2d CFT as a specific example of SymQRG, which preserves the Virasoro lines, the authors explicitly demonstrate how to rewrite the $T\bar{T}$-deformed partition function as a 3d gravitational path integral. Furthermore, they show that the deformed partition function could be written as the inner product between two states specified by physical boundary condition and topological boundary condition. The relation between Wheeler-DeWitt wavefunctional and $T\bar{T}$-deformed partition function indicates that the topological boundary state satisfies the "no-flux" condition i.e., it realizes a ground state of the 3d TQFT.

An equivalent tensor network type representation of SymQRG, motivated by the Turaev-Viro state sum model and the Levin-Wen stringnet model, is provided in the manuscript. In this framework, SymQRG can proceed iteratively from the coarse-graining of the triangulations of 2d surfaces. The authors study AdS_3 pure gravity as an explicit example and provide a state sum model for Liouville CFT and the SymQRG flow driven by $T\bar{T}$ deformation on it.

The authors generalize this construction to generic CFTs by proposing the topological holographic correspondence. In this framework, different CFTs, which respect the same set of symmetries, could be realized by different topological boundary states of the same bulk maximal SymTFT. The different boundary states are specified by gauging different sets of non-invertible lines (anyon-condensation). This leads to a exact bulk dual of generic CFTs and their SymQRG flows. The authors further comment on the resolution of factorization problem in semiclassical gravity calculation from the perspective of this exact topological holographic duality.

This manuscript is well-written with a very intriguing and convincing proposal. The authors show abundant evidence and clearly demonstrate an adequate number of derivations. Therefore, I recommend this manuscript for publication in SciPost. Before the publication, I have a few comments and questions for the authors to address.

- As commented below the flow equation (4.26), the sign of the $T\bar{T}$ deformation parameter realized in the tensor network model is opposite to the sign of the $T\bar{T}$ parameter in the gravitational path integral i.e., in Equation (3.1). Provided that the spectrum of $T\bar{T}$-deformed theory associated with AdS space with finite cut-off tends to become complex, would this property gives some hints of potential modifications to the tensor network model?

- In this manuscript, the $T\bar{T}$ deformation is mainly discussed as one example of SymQRG. Suppose the symmetry we want to preserve is the topological symmetries generated by Virasoro lines, are there any other symmetry-preserving deformations besides $T\bar{T}$ which we could add into SymQRG? For the SymTFT based on a generic fusion category, is there a systematic way to find all possible symmetry-preserving deformations?

- In the tensor network formulation, the certain type of symmetry preserving deformation could be specified by the boundary state in BCFT. For instance, the $T\bar{T}$ deformation is associated with the "shrinkable boundary condition". How would other symmetry-preserving deformations be related to the boundary states in the tensor network model? Moreover, the labels of the boundary states come from the module category $\mathcal{M}_{\mathcal{C}}$, which depends on the choice of the Frobenius algebra $\mathcal{A}$. Different CFTs with respect to the same set of topological symmetries will be realized by different topological boundary conditions, so they correspond to distinct choices of $\mathcal{A}$. Would this imply that the symmetry-preserving deformations on different CFTs with respect to the same topological symmetries are in general different?

- The sandwich construction seems to also work in higher-dimensional spacetime. However, the proposed construction relies on the topological nature of 3d gravity. Provided that the pure Einstein gravity in higher-dimensional spacetime is not so simple as a topological theory, would this cause any obstructions to build this exact topological holographic correspondence?

- It is proposed that the boundary CFT provides us with a quantum measure to define a 3d gravitational path integral. The CFT selects the bulk geometrical and non-geometrical solutions to be included in the gravitational path integral. What would a non-geometrical solution look like? How should we interpret these non-geometrical solutions?