Diffusive hydrodynamics of hard rods from microscopics

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The paper reports on a theoretical exploration of the large-scale Newtonian dynamics of an infinite, integrable one-dimensional model system of identical hard rods, moving ballistically in between elastic collisions where they exchange their momentary velocities with colliding neighbors. The authors derive first-order diffusive corrections to the Euler description which differ from the irreversible Navier-Stokes generalized hydrodynamics description. Instead, these corrections are described by two coupled equations invoking a spatially coarse-grained (in one spatial dimension) two-point correlation function, with higher-order correlations being of $\mathcal{O}(1/l^2)$ small. Quite interestingly, the coupled equations are time-reversible, generating thus no information loss on the considered hydrodynamic scale.

While the paper is difficult to comprehend in its mathematica details, it includes interesting theoretical results relevant to more specialized readers interested in the fundamental understanding of the thermalization of Newtonian many-body systems, and the appearance of (irreversible) diffusive hydrodynamic behavior on a coarse-grained space-time level. While in principle I can recommend publication of the paper, I have a number of questions and concerns (listed below) which should be addressed by the authors. These are related to the highly artificial character and hence the physical relevance of the considered one-dimensional hard rod (Tonk gas) system:

(1) In this integrable model system, the initial velocity distribution is preserved at least without external forces. In view of the according lack of thermalization (if starting from a non-Maxwellian velocity distribution) and ergodicity, it is perhaps not surprising that to first order in $1/l$ the diffusive Navier-Stokes behavior is not recovered. In the paper, the authors should elaborate in some detail on this point. It seems to me that thermalization and time-irreversibility are absent in general even when (all) higher order powers in $1/l$ are considered.

(2) In the context of their present work, can the authors comment on the single-filing constraint for the one-dimensional rod system, which causes a sublinear (fractional) time dependence of the mean-squared displacement of the rod centers at long times?

(3) What precisely is the relevance of the present work regarding the (hydro-)dynamics of more realistic Newtonian hard-disk and hard-sphere systems in two and three dimensions, respectively, where single-filing is absent?

(4) In the line following Eq. (4), l is defined quite vaguely as the ratio between macroscopic and microscopic scales. A more precise definition is desirable.

(5) Second line of Section 1.1: What is the physical meaning of the annotated 'quasiparticles'?

(6) Eq. (5) includes the 'empirical' density $\rho_e(t, y, q)$ whose definition, however, is postponed to Eq. (17). What precisely is the intended meaning of 'empirical'?

(7) Can the authors assign a physical meaning to the jump at $x = y$ of the long-range correlation function in Eq. (13) and Fig. 2 ?

[8] A few misprints caught on the fly while reading the paper:

- Bottom line on page 9: "This, way particles ..." $\rightarrow$ "In this way, particles..."
- Line preceding Eq. (34): "... Euler scale trajectory" $\rightarrow$ "... Euler scale trajectory is"
- Second line preceding Eq. (59): "... fully time-reversibly" $\rightarrow$ "... fully time-reversible"
- Line 5 of Sec. 4.1: "The simple idea to ..." $\rightarrow$ "The simple idea is to ..."

- Line 6 of Sec. 4.1: "... evolve it for $t = 1$" $\rightarrow$ "... evolve it to $t = 1$"
- Second line preceding Eq. (38): "... that satisfies (36)" $\rightarrow$ "... that satisfy (36)"
- Eq. (55): " + +" $\rightarrow$ "+"

In summary, I recommend publication of the paper after the points listed above have been addressed.