Report on the paper *Diffusive hydrodynbamics of hard rods from microscopics* By F. Hubner, L Biagetti, J. De Nardis, B. Doyon.

The authors study the dynamics of a one-dimensional gas of hard rods, focusing in particular on the diffusive corrections to the ballistic macroscopic behaviour. They claim that the correct description of these corrections involves two coupled sets of equations: one governing the evolution of one-point functions (densities) and another for connected two-point correlations. Their main argument relies on the fact that the dynamics generates long-range correlations in the presence of density inhomogeneities. Previous derivations of diffusive corrections (so-called Navier–Stokes corrections) did not take such correlations into account.

The authors state (see Section 5, "Conclusions") that their derivation is not a rigorous mathematical proof, and should therefore be considered heuristic. However, I find too many mathematical imprecisions to be able to follow the argument in detail, even though the main motivation — that long-range correlations affect Navier–Stokes corrections — is sound. Below I list some of these imprecisions. I hope that another referee may be able to follow these heuristic calculations more closely. On the other hand, the agreement with numerical simulations suggests that the equations obtained may indeed be correct.

Comments:

- Page 4, Section 1.1: In stating the main results, the definition of the dynamics and the notation should be given beforehand. For example, in (5) what is ρ_e? How does this correlation depend on ℓ?
- Page 7, Eq. (14): This is not a probability measure on the configuration space and cannot be normalized except under overly strong conditions on β. You probably mean a Poisson field with intensity $e^{-\beta}$, which should be stated precisely. (In Section 4 you mention this point correctly.)
- Page 7, 3rd line after (17): Who is ℓ^0? Do you mean terms of order $\mathcal{O}(1)$?
- Page 7, Eq. (18): I do not see why you call this "large deviation scaling"; it appears instead to be a condition on the correlations of the fluctuation fields.
- Page 8, Assumption 2: If you include in the statement "In particular, this is satisfied for all time t...", it seems to be part of the assumption, whereas in Appendix D you show that it follows from the dynamics.
- Page 8, Assumption 3: I do not understand this assumption. Correlations are always symmetric by definition. What is meant by "the microscopic shape"?
- Page 12: It is not clear why (36) and (32) imply (37), since derivatives in x are involved in (32). As the work is not a rigorous proof, this could perhaps be introduced as a reasonable assumption.

- Page 16, Eq. (60): With this density, the positions are not distributed according to a homogeneous Poisson process, as stated. The distances between them are therefore not identically distributed, but depend on their locations (they are rather localized under the density (60)).