REFEREE REPORT
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Title: *Holographic tensor network for double-scaled SYK*

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The author presented an interesting tensor network model that reproduces the moments of the Hamiltonian and the two-point correlation function in the double-scaled SYK model. The work is engaging and provides a pedagogical discussion on how the proposed tensor model captures the chord combinatorics of DSSYK. The manuscript certainly merits publication after the author addresses the following points:

1. In the review section on DSSYK, the author did not mention the convention used for the disorder average. It appears the convention adopted is:

$$\left\langle J^2_{i_1\ldots i_q} \right\rangle = \frac{\mathcal{J}^2}{\lambda \binom{N}{q}}, \tag{1}$$

   with the dimensionful parameter $\mathcal{J}$ fixed by normalizing $\mathrm{Tr} H^2 = 1$. It would be helpful to include this in the review section to ensure all notations are properly defined.

2. Could the author clarify how Z_k in equation (3.6) relates to the normalization of the state defined in equation (3.3)? From the current definition, the norm squared of the state,

$$\langle P|P\rangle = \sum_{\{\tau_k\}} \langle W| \prod_{i=1}^{k} (\tau_i D + (1-\tau_i)E)|V\rangle^2, \tag{2}$$

   does not appear to equal Z_k^2. Clarifying this relationship would be helpful.

3. Could the author show more explicitly how the proposed tensor network reproduces DSSYK results with the correct q dependence? While a one-to-one correspondence is mentioned between Dyck paths and non-crossing pairings, it is not immediately obvious that the tensor network reproduces the DSSYK result, as a given Dyck path corresponds to multiple chord configurations. For example, the first diagram in equation (4.2) evaluates $\langle 0|a_-^2 a_+^2|0\rangle$, which includes contributions from both non-crossing and crossing diagrams. A more detailed explanation would strengthen the main claim of the paper.

4. The current work primarily focuses on the no-particle sector of the chord Hilbert space. However, recent developments have explored the inclusion of matter chords, e.g., [1, 2, 3], and corresponding progress on the gravity side, e.g., [4, 5, 6]. It would be valuable if the author could provide some perspective on how the present model could be extended to include particle insertions in the chord Hilbert space.

References

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[5] Andreas Blommaert, Thomas G. Mertens, and Shunyu Yao. The q-Schwarzian and Liouville gravity. 12 2023.

[6] Andreas Blommaert, Adam Levine, Thomas G. Mertens, Jacopo Papalini, and Klaas Parmentier. An entropic puzzle in periodic dilaton gravity and DSSYK. *JHEP*, 07:093, 2025.