Report on the manuscript `2507.16904`: "Dancing in the dark: probing Dark Matter through the dynamics of eccentric binary pulsars"

In this manuscript, the authors investigate the impact of different dark matter models on the orbital period evolution of pulsar binary systems. They focus on dynamical friction in eccentric orbits, extending earlier results for circular orbits in collisionless dark matter. Their analysis indicates that orbital eccentricity can substantially enhance the effect. The topic is interesting, and the calculations appear sound and well-motivated. Nevertheless, I have a few questions and comments that I would like the authors to address before recommending the paper for publication in SciPost.

Overall.

- **(Major)** The figures are too small, which makes their content difficult to read; I recommend enlarging them for clarity.
- **(Minor)** The *Introduction* (and subsequent sections) states that "the local DM density in the Milky Way halo is typically $0.3 \pm 0.1\,\mathrm{GeV\,cm^{-3}}$." To avoid ambiguity, the authors should clarify that this value refers to the local density in the Solar neighborhood.

Introduction.

- **(Minor)** The literature review would benefit from a clearer distinction between studies addressing purely gravitational probes of dark matter and those invoking non-gravitational (direct) couplings. For instance, Refs. 24 and 26 predict significant observable effects only in the presence of direct scalar couplings. To strengthen the *Introduction*, it would be useful if the authors also highlighted the works (if these exist) that focus exclusively on gravitational effects and predict potentially observable signatures in pulsar binaries.

Dynamical evolution of binary pulsars.

- **(Major)** In arriving at Eq. (9) from $\mathrm{d}p/\mathrm{d}f$ and Kepler's third law, there should be a contribution of $\mathrm{d}e/\mathrm{d}f$; in particular, this includes a term in the radial component of the force $\mathscr{R}$. Could the authors clarify why does it not appear in Eq. (9)?
- **(Major)** Just after Eq. (27): "the expression (26) is valid only. . . ". It should be the opposite of what is written. Eq. (27) is valid when the de Broglie wavelength is much smaller than the cutoff radius (the semi-minor axis, in this case).
  For binary pulsars with $P_b \approx 100$ days, the separation is $\sim 10^{11}$ m. This is smaller than most of the Compton wavelengths considered in Fig. 5, and much smaller than the corresponding de Broglie wavelengths. This suggests that Eq. (27) is being applied outside of its regime of validity. Could the authors clarify that?
- **(Minor)** The manuscript adopts a Coulomb logarithm value of $\lambda \approx 20$. Could the authors clarify the origin of this estimate? Is the implicit choice of $b_{\max}$ consistent with what is used later in Eq. 25? A short discussion or citation would help the reader understand the basis for this choice and ensure consistency across the analysis.

Changes in the orbital period due to Dark Matter.

- **(Minor)** At the end of p. 8, the statements "increasing v_w for fixed σ generally leads to slightly higher $\dot{P}_b$" and "variations in σ for fixed v_w have a subtler effect" are unclear, since Fig. 1 appears to shows the opposite trend.
- **(Minor)** The authors state that $\dot{P}_b$ is positive both for ultralight dark matter and for certain regions of parameter space in the collisionless dark matter case. This seems counter-intuitive, as one would generally expect dynamical friction to produce orbital decay. Could the authors elaborate on the physical mechanism underlying this behavior? The manuscript would benefit from a brief discussion of this point.

Final remarks.

- **(Major)** The authors present estimates for the order of magnitude of the effects studied here and compare them with orbital decay from gravitational-wave emission. However, the discussion does not mention the typical sensitivity of current (or planned) pulsar timing observations to $\dot{P}_b$. Including this would help the reader assess the observational relevance of the predicted effects.