

The manuscript “Immobile and mobile excitations of three-spin interactions on the diamond chain” presents an analytical investigation of the localized and mobile excitation in a novel one-dimensional quantum spin model with three spin excitations. Motivated by the properties of so-called fracton excitation, the authors constructed a one-dimensional diamond chain model with three-spin XX-type interaction and transverse magnetic field. The model possesses a macroscopic number of local conserved quantities as well as a topological global one. Using duality transformation they mapped the initial model onto transverse-filed quantum Ising chain with alternating bonds and possible bond delusion given by the values of local conserved quantities. Diagonalizing analytically the emergent spin chain model the authors presented a detail analysis of the properties of mobile and localized excitation, which in their turn are topologically protected by the values of local conserved quantities. As the main results the authors determined the phase transition driven by the mobile excitations; demonstrated the link between local conserved quantities of the initial model and the number of localized modes in the emergent transverse-field quantum Ising chain and its segmentation; presented analytical results for one and two localized excitations, and calculates the Casimir-like force between them. The manuscript is scientifically sound and presents novel, non-trivial and interesting results. In my opinion, the manuscript can be published after the authors address several minor comments:

- p.2, Beginning of Sec.2, The authors referred to their model as a ”quasi-one-dimensional”, though it is exactly one-dimensional.
- p. 3, Fig.1, The illustration of the structure of the chain is a bit strange. It looks like the site  $C$  doesn't link directly to the sites  $A$  and  $B$ , and the three-links junction between them actually do not contain any spin. I understand the idea behind that, the authors want to emphasize the structure of the model after a duality transformation leading to just a chain with alternating bond  $a$  and  $b$ . But, in my opinion the conventional picture of a diamond chain, a corner sharing squares, is more relevant here. The emergent quantum Ising chain structure can be depicted as an additional dashed horizontal middle line with bonds  $a$  and  $b$ .
- The system has the symmetry with respect to permutation of  $A$  and  $B$  spins at each dimer part. So, the topological global symmetry operator given in Eq. (4) can be also constructed replacing  $\sigma_{i,A}$  with  $\sigma_{i,B}$ . Doesn't it responsible for additional symmetry ?
- p.6, The periodic case corresponding to symmetric sector after diagonalization (Bololi-ubov transformation) lead to only two bands, while the matrix  $M$  is 8 by 8. This means that each eigenvalue is fourfold degenerate. If there any physical symmetry behind this degeneracy ?
- Appendix B, The authors consider the system with different magnetic fields  $h_1$  and  $h_2$  on the single and on dimer sites. How does  $i$  affect the conserved quantities ?
- Although, the authors declared the direct link between their model and fractons in higher dimensional models, they did not demonstrate it explicitly or by any formal evidence.