

Title: The unitary representations of the Poincaré group in any spacetime dimension

Authors: X. Bekaert and N. Boulanger

This is an important review paper devoted to field realisations of the unitary representations of the Poincaré group in diverse dimensions. Its first version appeared in the hep-th archive in 2006:

<https://arxiv.org/abs/hep-th/0611263>

Since then it has generated almost 100 citations, which shows that many members of the higher-spin community have used this review in their research. As compared with the 2006 preprint, the current manuscript contains two new appendices A and B, as well as additional references. These additions are natural and useful.

I do not hesitate to recommend this review for publication. However, I have a few minor suggestions that the authors might consider to implement or not. (All suggestions are optional.)

1. Since the title of the review is “The unitary representations of the Poincaré group...”, it would be good to give explicit expressions for the positive definite and Poincaré-invariant inner products in the spaces of fields considered in the paper, in the spirit of the Bargmann-Wigner approach in four dimensions.
2. In four dimensions, one and the the same massive spin- s representation can be realised in several different spaces of on-shell fields of Lorentz type $(p/2, q/2)$ with $p + q = 2s$. It would be good to clarify how this works within the approach advocated in the review.
3. Recently, Ref. [36] put forward a covariant algebraic approach to describe the massless helicity representations of the Poincaré group in $d > 4$ using higher-dimensional generalisations of Wigner’s operator equation $\mathbb{W}_a = \lambda P_a$, where \mathbb{W}_a is the Pauli-Lubanski vector, P_a the energy-momentum operator, and λ the helicity of a massless particle. The authors might consider to give a few comments on this approach described in section 3 of [36].

I have several literature comments. First of all, I believe that the three-dimensional equations (94) appeared for the first time in the following publications:

- I. V. Gorbunov, S. M. Kuzenko and S. L. Lyakhovich, “On the minimal model of anyons,” *Int. J. Mod. Phys. A* **12** (1997), 4199 [arXiv:hep-th/9607114 [hep-th]]
- I. V. Tyutin and M. A. Vasiliev, “Lagrangian formulation of irreducible massive fields of arbitrary spin in (2+1) dimensions,” *Teor. Mat. Fiz.* **113**, 45 (1997) [*Theor. Math. Phys.* **113**, 1244 (1997)] [hep-th/9704132]

See eq. (3.26) in hep-th/9607114 and section 2 in hep-th/9704132. To the best of my knowledge, the equations (94) had not appeared in the earlier literature on anyons.

Further comments concern the last sentence in section B.2.1: “This system of equations can be generalized to the AdS3 background and be supersymmetrized, see [38] and references therein.” Specifically, my comments concern the supersymmetric extensions of Eqs. (94).

Supersymmetric extensions of (94) to AdS3 were given for the (1,0), (1,1) and (2,0) supersymmetry types in section 7 of the following work:

- S. M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, “Higher derivative couplings and massive supergravity in three dimensions,” *JHEP* **09** (2015) 081 [arXiv:1506.09063 [hep-th]].

In the case of N -extended Poincaré supersymmetry in three dimensions, supersymmetric generalizations of (94) were given for the $N = 1$, $N = 2$ and $N > 2$ cases, respectively, in the following works:

- S. M. Kuzenko and M. Tsulaia, “Off-shell massive $N=1$ supermultiplets in three dimensions,” *Nucl. Phys. B* **914** (2017) 160 [arXiv:1609.06910 [hep-th]].
- S. M. Kuzenko and D. X. Ogburn, “Off-shell higher spin $N=2$ supermultiplets in three dimensions,” *Phys. Rev. D* **94** (2016) 106010 [arXiv:1603.04668 [hep-th]].
- E. I. Buchbinder, D. Hutchings, J. Hutomo and S. M. Kuzenko, “Linearised actions for \mathcal{N} -extended (higher-spin) superconformal gravity,” *JHEP* **08** (2019) 077 [arXiv:1905.12476 [hep-th]].