Referee's report on the manuscript titled "Relativistic Landau quantization in non-uniform magnetic field and its applications to white dwarfs and quantum information"
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This work addresses the Landau Quantization (LQ) of electrons in non-uniform magnetic fields and its applications to problems cutting across different disciplines of physics, for example equation of state (EoS) and mass-radius (M-R) relationship of white dwarfs (WDs) as well as the quantum speed of electrons which might be important for processing the speed of quantum information. The non-uniform field has a simple power law variation as given by $\mathbf{B} = B_0\rho^n\hat{z}$ where n=0 represents the uniform magnetic field.

It is a well known fact that the zeroth Landau level is singly degenerate whereas it is doubly degenerate for all other levels in a uniform magnetic field. Authors in this manuscript demonstrate that the degeneracy of energy levels as found in a uniform field, is lifted in non-uniform magnetic fields. They further note that there is a definite alignment of energy levels of spin-up and spin-down electrons as the magnitude of a non-uniform magnetic field changes at each point. These are new findings. It is also observed that the quantum speed of electrons increases as 'n' grows.

The impact of the LQ of electrons in non-uniform magnetic fields is investigated on the EoS and M-R relationship of magnetized WDs. A specific example of the EoS for n=-0.3 is used to compute the M-R curve in Figs. 9. If one carefully compares Fig. 9(a) with Fig. 8(a), it is observed that the EoS of Fig. 8(a) for n=-0.3 is distinctly stiffer with respect to the case of the uniform field (n=0), but it is not the case in Fig. 9(a). This needs an explanation.

It is observed in Fig. 8(a) that less number of Landau levels is populated when n=-0.7 resulting in a softer EoS at lower densities and a stiffer EoS at higher densities. It would be worth demonstrating the effect of the non-uniform magnetic field for n=-0.7 on the M-R relation in a new plot.

In the introduction, authors mention another alternative approach of the non-uniform magnetic field in Ref.[27] extensively used in the investigations of structures of neutron stars and WDs. Authors should compare their current findings on the M-R relation of WDs with their previous results following the prescription of the magnetic field variation of Ref. [27] inside the WDs. This would definitely indicate whether new findings related to WDs in this manuscript are quantitatively significant or not.