Referee report for manuscript entitled “Exceptional points and pseudo-Hermiticity in real potential scattering” by Loran and Mostafazadeh submitted to SciPost Physics

In this manuscript, the authors use a novel scattering-matrix formulation to study scattering by real two-dimensional potentials, which have compact support in one dimension and are infinite in another. They find that exceptional points emerge quite generally in a pseudo-Hermitian operator associated with the transfer matrix of the scattering problem. The authors show an example of a set of exceptional points in a passive system - without gain or loss. The present work is thorough and rigorous and complements the authors’ previous work on this topic. The subject of exceptional points is interesting and timely, and finding passive systems that can be used to demonstrate their existence is advantageous. The weaknesses of the manuscript are, in my opinion, mainly its writing style and the lack of clear discussion of the implication or significance of the results. The manuscript contains many mathematical definitions and equations that are given without highlighting their physical meaning. I believe that condensing the text a bit and moving some of its derivations to the appendices, along with presenting some graphical illustrations of the results could improve the presentation significantly. My recommendation is that the paper should be revised significantly before acceptance because it is difficult to appreciate many of the presented results.

Specific questions and comments:

1. Are the exceptional points that you find related to the exceptional points associated with total internal reflection?
2. The authors use their recently developed scattering matrix formulation. I find its presentation difficult to follow. Since the formulation was published elsewhere, I recommend that the authors take this opportunity of writing a sequel paper on the same method to describe it in in an overview manner -- revealing the logic and essence of the approach. The authors can state the main equations one should use in order to construct the relevant operators in order to apply the method – rather than derive many results that were already published and present more than 100 equations.
3. I think it would be good to mention the advantages of the current formulation over previous scattering-matrix formulations of two-dimensional problems that require integration over momenta.
4. In Eq. 12, is p_0 defined anywhere? I think it is the incident momentum but could not find its definition.
5. The paper defines many quantities without explaining their physical meaning. To facilitate the reading, perhaps the authors can add explanations. For example, consider adding after the definition in Eq. (26) that it is an expansion of the identity operator in momentum space with weights w(p).
6. Is the number of real eigenvalues of H_hat (on page 8) equal to the number of bound states in the waveguide? Do the complex-conjugate pairs correspond to the continuum of unbounded solutions? What is the physical significance of their imaginary components? (penetration depths? lifetimes?)

7. I recommend plotting some of the results, perhaps using the studied numerical example. For example, the authors could plot the eigenvalues and eigenfunctions, showing that the spectrum contains exceptional points. Additionally, the authors could plot the dependence of scattering amplitudes on the geometrical or material parameters. Any plot of this sort and its discussion could have helped me appreciate the findings.
8. After Eq. 141, the manuscript makes an interesting point: "The presence of an exceptional points contributes terms … that correspond to the presence of terms in the reflection and transmission amplitudes that are rational functions of the length of the waveguide." Perhaps it could be mentioned in the abstract + introduction? If the authors plotted these reflection coefficients as a function of a parameter that drives the system into and out of the exceptional points, could they explain the behavior of the scattering amplitudes near the EPs I light of their analytic results in a graphical manner? Maybe plot these coefficients as a function of the length of the waveguide?
9. In the discussion, the authors say: "In particular, for an empty waveguide, we have shown that at the exceptional wavenumbers, where H develops an exceptional point, the transmitted wave includes a term that is independent of the length of the waveguide." Does this refer to the same point as my previous remark or is this a different result? Perhaps the authors could plot the contribution to the transmission that is independent and the one that depends on the length, and show how they relate to each other?
10. The authors say in the concluding paragraph that the addition of loss or gain would make the EPs disappear. Could they explain why this happens?