

The authors discuss an extension of the Standard Model (SM) usually referred to as 2HDM+a consisting of an extension of the scalar sector of the SM by an extra scalar doublet and an extra singlet (CP-odd). The model also includes a DM candidate in the form of a Dirac fermion, a singlet under the SM gauge group. The goal of the paper is to understand how specific processes constrain the model especially in the scenario where the pseudoscalar is very light. There were previous studies dealing with the same issue, like for instance 1802.02156, where one of the present authors participated. Two benchmark scenarios are used. I believe the paper is interesting although according to the scope of the paper it should be moved to SciPost Physics Core as it is not groundbreaking work. There are a few points that should be clarified before publication:

- The paper seems to very much built around the idea that eq. (4) makes a significant difference in the interpretation of the results of the searches. So my question is: had you included all constraints and all searches but not (4) would you get a sizeable difference in Figs. 1 and 2, left? Do the plots presented complete in any way what was presented in 1802.02156 by extending the mass region and by including new searches?
- Related to the previous comment, a short discussion about what happens to Figs. 1 and 2 left, if we move away from the benchmark points, is needed. I see that most of the light masses are excluded (except in regions where searches were not performed). Is this result robust against variation of the parameters?
- The authors state in the end of page 2 "For values of $|g_{haa}|$ that are not fine-tuned"; what does this sentence mean? And in the same paragraph can the authors please explain how the invisible Higgs decay width sets a lower bound of 100 GeV in the a mass? Or did I misunderstand the sentence?
- When the authors start discussing benchmark I, they write "This corresponds to a parameter tuning of around 5 %". Can the authors explain the meaning of this sentence? Same question for benchmark 2.

I believe that the paper should be published in SciPost Physics Core after these issues are clarified.