Remarks by Section

Abstract:

In the abstract there are some statements that are not quite correct in the level of generality they are stated. For example, Frobenius-Schur indicators can only be defined if a tensor category has a pivotal structure, and such a pivotal structure must be spherical if the Frobenius-Schur indicators are to take values in {\pm 1}.

While unitary (or anti-unitary) tensor categories are spherical, it isn't until Section IIA that the authors restrict their attention to unitary TQFTs (by which they mean unitary fusion and unitary modular fusion categories). The errors mentioned above can be avoided if the authors restrict their attention to unitary (and anti-unitary TQFTs) beginning in the abstract and Section I.

Section I:

I am quite partial to the definition of a QFT as an algorithm or set of rules to compute scattering amplitudes to complex numbers, and to the working definition of a TQFT given by the authors as "nothing more than a diagrammatic calculus that maps from the input diagram to the output amplitude".

However, getting the math and the physics correct requires more rigor about which (1) diagrams are under consideration (2) how the map from diagrams to amplitudes depends on the data of a pivotal fusion category.

The introduction and the remainder of the paper contains a rather serious mistake, which is the confusion of the diagrammatic representation of one of the "rigidity" axioms in a strict fusion category (their Figure 2) with the picture in Figure 5 depicting a different but closely related trivalent "zig-zag" diagram in a skeletal unitary fusion category. These are different pictures in different (equivalent but no isomorphic!) categories and the authors have perhaps become confused between the gauge freedom in choosing a normalizations in the creation and annihilation morphisms with the Frobenius-Schur indicator as a result. In fact Figure 2 is meant to agree with their Figure 10. In any case, the authors have misunderstood and misrepresented some important – and to be fair, subtle – aspects of unitary fusion category theory.

Section II:

There also seems to be some confusion about the fact that the Frobenius-Schur indicators and quantum dimensions are invariants of a spherical fusion category. It is not quite right to say the FS indicators are properties of anyons or that one can "define the diagrammatic loop weight d_a of a particle". These are invariants of a spherical fusion category that depend on the spherical structure. In fact, while the formula they give for d_a gives the right answer in a unitary fusion category because it will always be positive, it is not strictly speaking correct. The formula for deriving d_a depends on the Frobenius-Schur indicator! (The Frobenius-Schur indicators agree with the pivotal coefficients i.e. the constants that encode pivotal structure in the skeletal case in the unitary case.) The authors reference [20] but do not accurately reproduce its discussion of Frobenius-Schur indicators, which remains a authoritative source in the field.

Section IIA is described as a review of the definitions of “diagrams, F-moves, etc.”. The results of the paper depend on the definition of a skeletal unitary fusion category and their autoequivalences, but in trying to review these definitions and derive these from first physics principles the authors have made a few serious mistakes. The authors also use the phrase “braided theories” several times when they need to take seriously unitary topological system modeled by a unitary modular fusion category.

If Appendix B was intended to address these details, I could not verify e.g. whether the derivation of turning-up and turning-down operators was a correct derivation. At the very least the equations don’t correspond directly to the formulas for the action of turning up/turning down operators given in reference [20] in terms of the F-symbols FS indicators, and quantum dimensions.