

The authors study the boundary behavior of a 2d spin-1/2 model, whose bulk is tuned to a quantum critical point. They find a boundary phase transition as a function of boundary spin coupling between the ordinary boundary phase and an extraordinary boundary phase. They claim that the latter phase supports long-range antiferromagnetic order. Both the universality class of this transition and the nature of the extraordinary phase are claimed to be distinct from those in a classical 3D $O(3)$ model. This is a surprising finding that runs counter to the theoretical arguments presented in Ref. 15. If correct, it will be of interest to large swaths of condensed matter, statistical mechanics and quantum field theory community, as it would be the first known example of spontaneous breaking of continuous symmetry on the 1d boundary of a 2d quantum critical state. However, I would invite the authors to present a more detailed analysis supporting their conclusions. In particular, I believe it will be useful for the reader to see:

- A figure showing the correlation length (Eq. 4) as a function of system size in the extraordinary phase.
- A figure showing the Binder ratio as a function of system size in the extraordinary phase.
- A figure showing the Binder ratio in the vicinity of the special transition, and an analysis of the location of the special transition and y_s repeated using the Binder ratio.
- An analysis of the exponent η_{\perp} at the special transition - this can be used as a consistency check on the estimate of η_{\parallel} .
- An analysis of the boundary critical properties at some intermediate value of $J_s < J_{s,c}$ to confirm the ordinary nature of the boundary in this regime.

Also, I would like to point out that the critical value $J_{s,c} \sim 6.38$ reported by the authors is quite large and the simulations of the extraordinary phase are performed at even larger values of J_s . At such large values of J_s , ignoring the coupling of the boundary spin chain to the bulk, the boundary velocity is much larger than the bulk velocity. This velocity difference can potentially lead to slow cross-overs in the boundary behavior. In particular, such velocity anisotropies were considered in Ref. 15. It was shown that while in the extra-ordinary-log phase the surface velocity eventually flows to the bulk velocity, this flow is logarithmically slow and the exponent of the logarithmic fall-off q_{\parallel} of the correlation function depends on the surface velocity. This could potentially explain the dependence of the exponent q_{\parallel} reported by the authors in table 3 on J_s , although it does not immediately explain the difference of q_{\parallel} extracted from C_s and $S(\pi)$. Ideally, the authors would address the surface velocity by studying correlations along the time-direction. However, if their numerical algorithm does not easily allow for this, they should at least comment on the issue in the paper.