Report on

Spin degrees of freedom incorporated in conformal group: Introduction of an intrinsic momentum operator

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This work is an attempt to introduce "intrinsic momentum operators π_μ", which do not exist in an ordinary context, to conformal group. For the sake of simplicity, the author restricts himself to $(3+1)$-dimensional Minkowski space and gives π_μ explicitly for relativistic wave equations with spin $s = 1/2, 1$ and $3/2$. To be consistent with the scalar nature of fields, the author places a constraint on π_μ such that π_μ annihilates physical states.

The issue discussed is an interesting problem worth investigating. The presented results are new and plausible so that worth publishing. However, there are some minor issues which should be clarified before accepting for publication.

(i) In Tables 1 and 2, equations with mass term are presented. In general, massive theories are not scale invariant so they are not able to be conformal. It should be clarified which equation, massive or massless, is considered. Or which definition of conformal symmetry is employed.

(ii) Physical states are defined by $|\psi_{\rm ph}\rangle = \mathsf{P}_k |\psi\rangle$ where $k = 1, 3$ for spin 1 and $k = 1, 4$ for spin 3/2. It is easy to verify that $\pi_\mu \mathsf{P}_1 |\psi\rangle = 0$, however, it seems that $\pi_\mu \mathsf{P}_k |\psi\rangle = 0$ dose not true for $k = 3, 4$. The same problem is also observed for $s = 1/2$ Dirac equation, i.e., $\pi_\mu \mathsf{P}_1 |\psi\rangle = 0$, but $\pi_\mu \mathsf{P}_2 |\psi\rangle \neq 0$.

(iii) Perhaps, the author may have written the manuscript in haste. There are some sentences with unclear meanings and grammatical errors. It is recommended to polish the English sentences before publication. Some examples are in order:

- p.2, the last line:
  the invariance of $\epsilon^{\mu\nu\rho\sigma}$ under (6) is not so trivial a matter $\rightarrow$ trivial
- p.2, 1st line of §3:
  This section deal $\rightarrow$ deals
- p.2, 2nd line of §3:
  which satisfies (18) $\rightarrow$ Is the equation number correct ?
- p.5, below eq.(19):
  In may be convenient $\rightarrow$ It may be...
- p.6, below eq.(23):
  Note that by (22) $\rightarrow$ Is the equation number correct ? The referee do not understand how (22) concludes $\{X_{\rho\nu\mu}, s_5\} = 0$ and equation (24).

- p.6, 2nd line after eq.(24):
  For example, we have $Y^{(\frac{1}{2})}_{\rho\nu\mu} = \cdots$ $\rightarrow$ Is the equation $Y^{(\frac{1}{2})}_{\rho\nu\mu} = \cdots$ an example of the text before ? It was difficult to see (at least for me) the connection with the previous texts.