Manuscript Number: scipost_202301_00037v1
Title: Floquet engineering non-equilibrium steady states

Report

The manuscript presents the use of a numerical trick to compute the gradients of the floquet steady state with respect to the control parameters. This approach allows to reformulate floquet engineering as an optimization problem where gradient-based methods can by applied. Over all, the paper presents a contribution to the quantum optimal control theory field. I find the manuscript well written and clear. After addressing the following comments and questions, the draft could proceed to publication.

Questions:

- The goal of the paper is to maximize observables that can described with Eq 6 by modifying field parameters using gradients. The computation of $G(u)$ depends on the time trajectory, which could be also computed using the adjoint sensitivity method. I encourage the authors to include this in the discussion of their work.
- As indicated by the authors the most similar paper is Ref. 28, where it was previously demonstrated the possibility to evaluate the jacobian of an ODE's steady-state with respect to the parameters that govern the dynamical evolution. The presented work expanded this idea to time periodic systems, however, there is a lack of notation and clarity.
- Are Eqs. 15-17 similar to Eqs. 4-5 from Ref. 28? If so, the authors should indicate it and explain its differences.
- I believe the following term is ambiguous, "Although to our knowledge, no previous work has attempted the optimization of NESSs with respect to the external drivings, a related work [28] ... ". Parameters that describe the system and control parameters globally affect the system so, under Ref 28's framework, one could also compute such gradients with respect to the control fields.
- Why was sequential Least-Squares Quadratic Programming algorithm the only used optimization protocol? Briefly discuss if this method present an advantage over other more standard ones, e.g. BFGS or other.
- I find the title misleading as engineering of quantum states could also be done with grid or sampling based approaches. The core result of the paper is an algorithm to compute gradients for the floquet steady-state, why not include that in the title of the paper?
- Increase the font size.
- Regarding the normalization condition, $\mathrm{Tr}\rho_u = 1$. For Lindblad-type systems, the Liouvillian is trace preserving, which indicate that $\mathrm{Tr}\rho_u = 1$ is an unnecessary constrain.
- Include a figure that shows the convergence of the search problem over the number of iterations. Additionally, are the field parameters initialize randomly?

Comments:

The authors are missing some relevant citations,

- Phys. Rev. Research 4, L012029 2022
- arXiv:2011.12808
- Lev Semenovich Pontryagin, EF Mishchenko, VG Boltyanskii, and RV Gamkrelidze. The mathematical theory of optimal processes. 1962.
- The Implicit Function Theorem: History, Theory, and Applications (Berlin: Springer), S G Krantz and H R Parks (2012)