Report on the paper scipost 202304−00018:

Flux Vacua and Modularity for Z_2 symmetric CY manifolds

by P. Candelas, X. de la Ossa, P. Kuusela and J. McGovern

The authors aim to provide evidence in support of a conjecture of Kachru, Nally and Yang concerning the modularity of flux vacua. This is an interesting question and the paper contains some extensive work in this direction. I find particularly interesting their foray in the Deligne conjecture, which has made an appearance in physics quite some time ago before the work on Yang, but which otherwise is rarely considered in physics. Also interesting is the consideration of the KNY modularity conjecture in the context of algebraic extensions of the rationals. I recommend the paper for publication but would like to make a few comments for the authors to consider.

Remarks:

1. I believe that there is a typo in the definition of j and that as written the Eisenstein series E_4 should be replaced by G_4. It seems that the authors use the Koblitz definition of the weight 12 form Δ_K, given by

$$\Delta_K \;=\; g_2^3 \;-\; 27g_3^2.$$

While classical, dating back to the 19th century, this definition does not have rational coefficients and is not normalized in the standard way via the Dedekind eta function as $\Delta = \eta^{24}$. Using this Koblitz definition of Δ_K, and adjusting the definition of the j-function given in the paper by replacing E_4 by G_4, i.e. using

$$j \;=\; 1728 \cdot 60^3 \frac{G_4^3}{\Delta_K},$$

leads to a more pristine form of the j-function as

$$j \;=\; \frac{E_4^3}{\Delta}$$

that now does involve the integrally normalized E_4, as well as the integrally normalized discriminant form $\Delta \;=\; \eta^{24}$. This latter form of j avoids the various factors of (2π) that arise from a lattice starting point. Irrespective of form in which the authors end up defining the j-function, and in view of the fact that different versions of the discriminant form exist in the literature, it would be useful to have the definitions of Δ and E_4, or G_4, handy, perhaps in the modular form appendix of the paper.

2. It would be useful to give the Sturm bounds for the modular forms that appear in the paper.

3. In the present paper and in previous work the authors proceed by computing the zeta function of a variety and then identify "motivic pieces" of the cohomology by factorizing the zeta function. It would be interesting, and more direct, to proceed in an alternative way by first attempting to identify the motives first and then computing the motivic L-functions. Work in this direction has appeared in the physics literature some time ago for certain families of CYs, although in a different physical context (Kadir et al, 1012.5807 [hep-th]).

4. Finally, I've noticed a few trivial typos on pages 14, 27 and 134.