Report on scipost_202309_0000v1
Invariant approach to the driven atom-field interaction

The Jaynes-Cummings model is a fundamental model in the study of the interaction between an atom, represented by a two-level system and a single-mode quantised electromagnetic field.

In the present paper, the Jaynes-Cummings model subjected to both, a classical field acting on the two-level system and a classical field acting on the quantised single-mode photon, is investigated and some of its dynamical variables are evaluated. The two classical fields drive the two-level system and the quantised field with a common frequency but different amplitudes. These dynamical variables are the atomic inversion and the average photon number. The simultaneous treatment of classical fields driving the quantum degrees of freedom of the Jaynes-Cummings model is the major novel aspect introduced by the present authors.

The paper is, on the whole, well written and documented with the appropriate references and clearly merits publication.

I suggest a few changes and amendments:

- The major change I suggest is in the title of the submission which I find to be too general. Since the manuscript deals exclusively with the driven Jaynes-Cummings model, not the atom-field interaction in general, I suggest that this is made clear already in the title by, e.g., changing "atom-field interaction" into "Jaynes-Cummings model".

- Due to developments in recent decades, a model going beyond the rotating wave approximation, the quantum Rabi model, also introduced by Jaynes and Cummings in the article cited of the present manuscript, has attracted considerable experimental and theoretical attention. The applicability of the quantum Rabi model is not restricted, as in the Jaynes-Cummings, to small atom-field interaction strength. It would therefore be appropriate to mention this model in the introduction of the present paper together with some key references. For example, in

  Niemczyk T et al 2010 *Circuit quantum electrodynamics in the ultrastrong-coupling regime* Nat. Phys. 6 772

  and

  Casanova J, Romero G, Lizuain I, García-Ripoll J J and Solano E 2010 *Deep strong coupling regime of the Jaynes-Cummings model* Phys. Rev. Lett. 105 263603

  larger values of the atom-field interaction strength have been explored.

  While the Jaynes-Cummings model can be solved by elementary means, this is not the case for the quantum Rabi model which has only been solved quite recently in

Braak D 2011 *Integrability of the Rabi model* Phys. Rev. Lett. 107 100401

using sophisticated analytical techniques.

- A minor typo: thing → think in the fourth line of section II.
- The explanation that Ω_n is now a number, not anymore an operator $\hat{\Omega}_n$ as in equations (17) and (18), should already appear earlier, after equation (20).
- An interesting quantity not calculated by the present authors, but presumably within the reach of the method presented, would be the variance of the average photon number.
- For clarity, the inset indicating the analytically calculated versus the numerically obtained results in figure 5 should also appear.
- After equation (26), it is said that the Rabi frequency Ω_n is modified, but no explicit expression is given. Do the authors refer back to equation (18)? Is this a statement with reference to figure 3?
- There are a few awkward phrases which should be taken care of, e.g.
  - as the operator has been already acted
  - being $\hat{n} = \hat{a}^\dagger \hat{a}$

  the latter appearing similarly in several places.