In this paper, the authors have analysed the energy inequalities or more precisely the null energy conditions in the context of non-minimally coupled scalar fields and gravity. It is well known that the null energy condition is violated even classically if the matter field is non-minimally coupled. Here the authors have taken a very particular case non-minimal couplings and interpreted this violation from many different perspectives. The key message in the paper as I have understood is the following.
Whichever way we would like to view the null energy condition, - either as a condition on the classical stress tensor at every space-time point or an average null energy condition generated by an integration along a null geodesic or a condition on the expectation value of the smeared stress tensor operator on a particular quantum state, - it is always possible to violate the condition. However, the violation would typically take place at large values of the field (or at large values of the expectation value of the field operator in the quantum case). Now if we assume the non-minimally coupled theory as an effective field theory, then it must be valid only below some cut-off and the violation could be ignored as being above this cut-off scale. Then in the last section the authors have given a heuristic path integral argument indicating a break down of the semiclassical approximation at large values of field where the one loop contribution to the path integral could be of the same order or higher as that of the leading term.

The analysis has been done in two different frames related by field redefinition. For this particular theory of gravity coupled to scalar, it turns out that the Lagrangian could be cast in a non-minimal form by redefining the scalar field and the metric. In this frame (called Einstein frame) at least classically the null energy condition is obeyed, whereas in the frame where the field is non-minimally coupled (called Jordon frame) it is not. There are some discussions about which frame should be considered more physical. This is the point I have some confusions which I am listing below.

1. The fact that the non-minimal coupling could be completely removed by field redefinition seems to be a very specific property of this particular non-minimal coupling. In fact all the conclusions including the value of the cut-off on fields also very much depend on the details of the Lagrangian. However, the final message that to deal with the violation of null energy conditions any non-minimal coupling should be viewed as an EFT with a cut-off on the field is a very generic one. I feel it would be nice to have some discussion on how the analysis could be extended to any general non-minimal coupling.

2. The definition of a field is a matter of choice and the final physical outcome should not depend on how we choose to define fields. I agree with the authors that the correct question is whether some exotic geometry could be generated or not and the answer to this question should not depend on which frame we choose to describe the geometry.

However, it seems that the invariance of physics under different choices of field variables is a very general principle, independent of whether we want to describe the physics classically or quantum mechanically, as an EFT valid only for small values of fields or as an exact theory valid everywhere. From this perspective I am confused if the null energy condition is classically not violated in Einstein frame, then why it is violated in Jordon frame at large values of the field. In other words, why the null energy (a physical quantity) expressed in terms of the fields in Einstein frame is always positive wheres the same quantity just rewritten in terms of a new field variable (this is what I mean by field redefinition) could have any sign.