Referee report for
Energy-saving fast-forward scaling

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I. FULL REPORT

The present study by Hatomura provides an interesting perspective on the fundamental issue of the costs of quantum computation. For the realization of quantum technologies, efficient and accurate time-dependent quantum control is essential. This becomes more challenging when control speed is increased in order to overcome the effect of decoherence. In this work, the author applies the theory of fast-forward scaling (FFS) to shorten the timescale of a system's dynamics without changing the measurement outcomes, and then investigates the corresponding instantaneous and total energy cost of such a process. The cost metric is taken to be the Hilbert-Schmidt norm of the total system Hamiltonian, thus accounting for the modulation of the Hamiltonian in time. With this, the author investigates a number of simple models of central importance, namely a two-level system (qubit) with both a time-independent and time-dependent measurement basis and quantum annealing in an Ising spin glass. It is found that optimizing the phase in the formulation of FFS enables one to reduce both instantaneous and total energy costs compared to a standard benchmark case. When full optimization is challenging, modulation of the phase has been shown to further reduce the energy cost of the dynamical evolution process. This promising result indicates that requiring fast manipulations of quantum systems does not have to necessarily come at the expense of energy efficiency. In this sense, the work opens a new pathway in an existing research domain to achieve fast dynamics in quantum devices without excessive energy overheads.

The manuscript is coherent, well-organized and algebraically sound. However, if the article is to target the broader community investigating shortcuts to adiabaticity and find applications in the field of quantum computation, some further clarification and discussion surrounding the central results is required. In particular, the analysis of the main analytic results and figures is somewhat lacking, reducing the impact of the work. The manuscript also does not outline any potential experimental implementation of results. Given the importance of energy saving and the timeliness of the research, it would be unfortunate not to elucidate the potential of this work on cold atom or solid-state platforms. In terms of grammatical and orthographic proficiency, some work is still required. Below I provide suggestions to improve the manuscript, and some points that I would like to see clarified. After the author has satisfactorily addressed these, I can recommend the manuscript for publication in SciPost Physics.

A. Remarks, Suggestions and Requested Changes

Italicized font highlights text that has been taken directly from the manuscript.

1. General comments

1. In Sections IV and V, I encourage the author to provide some additional details – either in the main text or in brief appendices – so that arguments and derivations can be easily reproduced by a reader. In most cases, a few more in-text references to relevant equations will suffice. When a result is stated for a slightly more involved calculation, a sentence providing further details on the intermediate steps, potentially with an additional equation, would greatly improve reproducability of the work.

2. It is recommended that the manuscript is proofread carefully, and that grammatical and orthographic errors are corrected, with a focus on the introductory section of the article. In addition to minor typographical errors, I suggest that the author corrects the spelling of all occurrences of the words 'speedup' and 'slowdown' to ensure that they have the appropriate part of speech. For example, 'speedup' and 'slowdown' in the abstract are verbs, and therefore should be written as two separate words. When 'speedup' is used as a noun the current spelling is correct.

2. Section-specific comments

Introduction

1. With the current phrasing and typographic error, the meaning of the following sentence is unclear: "*For sustainable development, limited resources should carefully be used with mitigating sacrifiction of convenience*".

2. In the second paragraph the author introduces shortcuts to adiabaticity, outlining various approaches. The paragraph closes with the statement that "*all these methods theoretically offer arbitrary speedup*". I believe that there are certain limiting factors, or upper bounds, imposed on the theoretical speedup one can attain, see for example literature on the topic of quantum speed limits. Since the work investigated in this manuscript is likely to be impacted by these unavoidable constraints on the physically achievable speedup, it would be good to include literature references here and further discuss any potential limitations of the results later in the manuscript.

3. The phrasing of the final sentence of paragraph three requires attention. Perhaps consider the following: "Our main finding is the existence of a nontrivial speedup of processes with reduced energy costs, which we propose as energy-saving fast-forward scaling."

Fast-forward scaling

1. The formulation of the fast-forward scaling theory (FFST) in the manuscript is done using inverse engineering approaches. From what I understand, the speed-controlled dynamics depends on two important quantities, namely the real, time-dependent phase $f_\sigma(t)$ and the scaled time $s(t)$. The latter is simply a naive scaling with respect to time, but also introduces the so-called *magnification factor* in some descriptions, see for example Ref. [13] of this manuscript. This factor provides a (quantitative) means to accelerate, decelerate or stop the dynamics. How does this factor impact the results presented in this manuscript, and is it possible to infer anything about energy costs when the dynamics is decelerated? From what I understand, the current investigation into a process' total energy costs relies on $ds/dt \geq 1$. It would be appreciated if this point is clarified and commented on, perhaps in Sec. III where a discussion would be most natural.

Energy costs of fast-forward scaling

1. The Hilbert-Schmidt norm of the total Hamiltonian is the metric used to analyze the energy cost. Given the centrality of this quantity to the results throughout the manuscript, I propose including a clear, concise definition of the norm using the notation of the article to improve readability and assist with the reproducibility of analytic expressions.

Energy-saving fast-forward scaling

1. With the information provided in this section, it is difficult to reproduce the second term of Eq. (11). In particular, how is the prefactor of the $\langle\sigma|\partial_t\sigma'\rangle$ term imaginary? From what I understand, this result is derived from the generic fast-forward Hamiltonian in (4), with the unitary operator given in Eq. (6) and measurement basis taken to be $\{|\sigma\rangle\}$. Including some further details, either in the main text or an appendix, is recommended, particularly since these expressions are central to the remainder of the manuscript. At the very least, it would be useful to include an in-text reference back to Eq. (4) before the expression for the norm $||\hat{H}_{\mathrm{FF}}(t)||^2$ is given (see Eq. (11)).

2. For the time-independent measurement basis, the minimization procedure with respect to the phase leads to a norm, $||\hat{H}_{\mathrm{FF}}(t)||^2$, that takes the form of a variance, see Eq. (13). In other settings, the variance of a Hamiltonian describes energy fluctuations and can provide information on stability. Can any of these interpretations be applied here and enable us to better understand the energy costs of these fast-forward scaling implementations? Also, it appears that the measurement basis $\{|\sigma\rangle\}$ cannot be the eigenbasis of the Hamiltonian $\hat{H}(s)$, as this would result in zero energy cost. I would like to see this case explained further, or, alternatively, a clarification that the basis should indeed be different from that of the Hamiltonian being considered (perhaps one can include this as early as Sec. II).

Examples

1. As mentioned under general remarks, in-text references and the addition of one or two concise sentences would improve this section. In Sec. VA I recommend linking back to the simplest FFST example after Eq. (21) when

$\omega(s) = 0$. For the arguments of the trigonometric functions in (22), please reference the phase in Eq. (20) and perhaps also aid the reader in observing that $f_{\pm}(t) = \pm \int_0^s ds' \, \omega(s')$. Moreover, is there a reason for stating the fast-forward Hamiltonian explicitly, both here and in section B? It appears that this is never used, making it redundant. If it is in fact important and assists in the overall interpretation of the result, I kindly ask the author to provide further information highlighting this.

2. In Sec. VB, it is written that "*we do not consider full optimization of the energy costs because it results in a complicated fast-forward Hamiltonian and it would be difficult to realize in experiments*". It may be beneficial to expand on this point, highlighting what exact trait makes the fast-forward Hamiltonian difficult to realize, and how the (sub)optimization performed for the quantum annealing combats these challenges. Are there other results for the (sub)optimal phase that may make the annealing even easier to realize experimentally, even if it is at the expense of a slightly higher overall energy cost? When considering the instantaneous cost, see Eq. (30), it would appear that the choice of measurement basis is very important. Working in the computational Pauli-z basis, the strength of the longitudinal field controls 'how close' one is to the standard cost. Is there a benefit to choosing a measurement basis based on the parameter that one has the ability to tune experimentally? Can one still achieve the same results? Naively, I would assume that fully exploiting the ease of tunability of specific parameters may allow for improved energy saving in experimental settings.

3. In light of reproducibility of Eqs. (32) and (33), I recommend including the expressions for the inner products $\langle E_+(s)|\partial_t E_-(s)\rangle$ and $\langle E_-(s)|\partial_t E_+(s)\rangle$. After Eq. (37), it is mentioned that δ is small in the phase modulation. What characterizes the smallness of this parameter? In other words, small relative to which other energetic or temporal scale?

Conclusions

1. While the conclusions drawn are clear and provide a complete summary of the questions investigated in the manuscript, there are no objective statements regarding the potential limitations or restrictions of the present study. In the case of this work, the attainable rate of change of control parameters for quantum devices can definitely provide limitations. Comments on this would make the main message of the article stronger. Also, there are instances where one may want to vary control parameters more slowly, enabling some experimentally feasible driving schemes. If the author feels it would be useful, I suggest mentioning how the present results may be extended to this case. Finally, to recommend the manuscript for publication, I advise that at least one or two perspectives are offered for potential future work.

3. Figures

1. Figures are neat and informative, but I find the discussion surrounding the figures, as well as the interpretation thereof, too minimalist and, in the case of the second figure, completely absent. Concerning the instantaneous energy cost in Fig. 1, it would be insightful to understand why the spike in the energy cost is not suppressed in a monotonic fashion with the increasing (constant) strength of the transverse field, Γ_0. Given this behavior, is it possible to determine the transition point in Γ_0 that separates regimes in which the instantaneous cost in the vicinity of the energy gap increases with (1) increasing transverse field strength and (2) decreasing field strength? A final curiosity regarding this figure is that if one were to use the instantaneous cost to ascertain the total cost of the fast-forward scaling procedure, then the total cost does not appear to increase monotonically with $2\Gamma_0$, as in Fig. 1 (right). Can this be explained?

2. Regarding the right subfigure of Fig. 1, a discussion on the limiting behavior, $\Gamma_0 \gg 1$, would be an informative addition to the manuscript. In the presence of strong transverse fields, does the energy cost keep increasing monotonically with the field strength? If this is indeed the case, then the total energy cost compared to that of the original process will eventually be such that $C/C_\$ > 1$, inferring that the fast-forward scaling protocol that has been proposed results in higher energy costs. Understanding this transition point may prove insightful for applications.

3. The second figure only displays one instance of the phase modulation applied for the scenario of a time-dependent measurement basis. To better interpret how sensitive the instantaneous energy cost peak is to the modulation δ, I encourage the author to include curves for various δ values.

4. Finally, for a reader to quickly reference the parameter values used, I would incorporate the information on

parameter values at the bottom of page 5 in the caption of Fig. 1. Similarly, the modulated phase also depends on quantities such as ω_0, T and T_{FF}, so it would be advisable to include the exact values in Fig. 2's caption for reproducibility.

II. REQUESTED CHANGES

The requested changes are clearly outlined in the referee report, with the list below only providing a brief summary for the author's convenience. For further details and context, please refer back to the main report.

1. Grammatical and orthographic errors should be corrected, with a focus on the introductory section of the article.
2. It would be beneficial to include a discussion on the limiting factors, or upper bounds, imposed on the theoretical speedup one can attain. How do these limitations impact the results of the manuscript?
3. Clarification on the impact of the so-called *magnification factor*. In particular, can the theory developed in the manuscript be applied to the case where the dynamics is slowed down? At present it appears that the result for a process' total energy cost relies on $ds/dt \geq 1$.
4. Inclusion of a clear, concise definition of the Hilbert-Schmidt norm using the notation introduced in the manuscript.
5. It appears that the measurement basis $\{|\sigma\rangle\}$ cannot be the eigenbasis of the Hamiltonian $\hat{H}(s)$, as this would result in zero energy cost. I would like to see this case explained further, or, alternatively, a clarification that the basis should indeed be different from that of the Hamiltonian being considered.
6. Sec. VB raised several questions about choice of basis and potential impact of using a different optimized phase, see main report. It would be appreciated if these questions could be addressed.
7. After Eq. (37), it is mentioned that δ is small in the phase modulation. What characterizes the smallness of this parameter?
8. Addition of in-text references to previous relevant equations in sections IV and V to assist readers in reproducing results in the manuscript. In some cases, the author should consider adding a sentence to make the derivation of certain expressions more transparent.
9. If possible, please address the questions/comments on the figures, with a focus on points 1 and 3. The remaining points are just suggestions that the author may incorporate should he/she wish.
10. In the conclusion some statements regarding the potential limitations or restrictions of the present study should be given. Additionally, some perspectives should be offered for future work.