REFEREE REPORT FOR *TOPOLOGICAL LINEAR RESPONSE OF HYPERBOLIC CHERN INSULATORS*

First of all, apologies to both the authors and the editors for my slow response.

The paper is well-written and clearly an important contribution to the still fairly young area of hyperbolic matterials. The two other referees have already asked most of the questions I meant to, below I focus on the two points I wish the authors have addressed. Otherwise the paper have my support for submission.

1. Questions

I have two main questions, but they are both special cases of the following broader one:

What about the finite ($G = \Gamma$) case?

While I understand the reasoning to choose periodic boundary conditions to make G finite, I think addressing this a bit more in depth would be useful, especially since in the euclidean case there are nice treatments of this.

Question #1: Maybe I am misunderstanding something here, but since [1] was not referenced, I wanted to be sure. Is $\Gamma_{\mathrm{PBC}} \subset \Gamma$ secretly a noncommutative generalization of $\mathbb{Z}_p \times \{\mathbb{1}\} \subset \mathbb{Z}^2$ / can *gauged*-translations be somehow defined as in [1]? of course this is meant in the special case when the flux through a fundamental domain of Γ_{PBC} is integer.

The broad summary of this question is:

Is there a connection to [1]?

I think such a point of view is 1. feasable, and 2. would make the connection to the euclidean case even nicer. Finally, in my opinion, it would make the result even more appealing to the mathematical physicists community.

On that note...

Question #2: This is sort of a continuation of the previous question, so if the answer that is somehow a firm "No", then please disregard it. But, in case, the setup can be recast as periodic system with "gauged translations" (again, as in [1, Equation (2.6)]), then can you recast the infinite system as theory over a noncommutative genus-g surface? Hints of such an interpretation can be found in eg. [2, Sections 4.3 and Section 5.]. Such an interpretation (that is, seeing QHE as a theory over the noncommutative torus) was very fruitful in the euclidean theory, so I belive a mention would interest the Reader (definitely would interest me).

Date: September 23, 2024.

References

[1] Mahito Kohmoto, *Topological invariant and the quantization of the hall conductance*, Annals of Physics **160** (1985), no. 2, 343–354. ↑1

[2] Ákos Nagy and Steven Rayan, *On the hyperbolic bloch transform*, Annales henri poincaré, 2024, pp. 1713–1732. ↑1