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Report

In this paper, the authors use the MPS of the CBA to write the unitaries of ABCs for the inhomogeneous XXZ model. They demonstrated that the exact unitaries can alternatively be obtained by performing a basis transformation in the auxiliary space of the ABA.

I believe it deserves to be published in Scipost after the following comments have been appropriately addressed:

1. **Reviewer**: Above Eq(3.27), the authors state " (3.25) realizes a Bethe wave functions with M magnons propagating over N spins the inhomogeneous spin chain, we identify it the...". In Section 3.3, they state" Having obtained the MPS of the CBA (3.25), we are in the position to construct ABCs for the inhomogeneous spin chain..." It appears that Eq. (3.25) is crucial. Since the Bethe state should be an eigenstate of the inhomogeneous spin chain, could the authors provide some numerical verification of this equation for small lattice sizes?

2. **Reviewer**: Above The Table 2, the authors state "However, this knowledge does not mean can evaluate scalar products efficiently in general; rather, the limitation must be taken into account in the numerical computation of the unitaries", Could they elaborate further on what this "limitation" refers to and how it should be addressed? Additionally, the tables should be made clearer. For instance, what do "domain" and "image" mean in Tables 1-3?

3. **Reviewer**: In Section 3, the number of qubits in which the unitaries act defines two classes: long and short unitaries, and they summarize the properties of long and short unitaries. So, what will happen if the number of spins N becomes infinite? It would be nice to provide some physical interpretation for this.