Karlsruhe Institute of Technology

Dear Prof. Wen:

This is my report for Palmerduca and Qin: "Four no-go theorems on the existence of spin and orbital angular momentum of massless bosons".

I have read the article with interest. I find it well-written, timely, and important, because they treat a confusion that affects different areas in physics, in particular and prominently, optics. I find most of the arguments precise and I agree with most of the conclusions. However, I disagree with some parts, which I list below, and they should be addressed before proceeding to the publication of the article. I am convinced that clearing these points will reinforce the important message carried by this work.

Best regards,
Ivan Fernandez-Corbaton

1) I disagree with a statement that is written in the abstract

" SAM-OAM splitting is unambiguous for massive particles",

and also in the introduction

" For massive particles, the angular momentum naturally and uniquely splits into SAM and OAM.",

and whose justification is contained right after Eq. (17).

I do not think that the split is valid even for massive particles. Here is why.

For a particle of mass M, it is clear that the little group of the standard vector with four-momentum (M,0,0,0) is SO(3). Note that (M,0,0,0) represents a particle at rest with 3-momentum equal to (0,0,0). For such particle at rest, the spin-1 matrices are indeed the angular momentum operators.

However, this does not mean that the spin-1 matrices are good angular momentum operators for the general case of massive particles out of their rest frame. Rather, one can see in the definition of **J**

$$\mathbf{J} = \mathbf{r} \times \mathbf{P} + \mathbf{S} \tag{1}$$

that $\mathbf{J} = \mathbf{S}$ when $\mathbf{P} = \mathbf{0}$, as a particular case.

The point is that the Poincare group contains only one kind of spatial rotations, and such transformations have $\mathbf{J}$ as their generators.

I refer to the authors to Sec. 10.4.2 of Wu-Ki Tung's book: Group theory in physics, and also to to Sec. 16 of the fourth volume of the Landau and Lifshiftz course of theoretical physics: Quantum Electrodynamics, where it says that: *"In the relativistic theory the orbital angular momentum L and the spin S of a moving particle are not separately conserved. Only the total angular momentum is. The component of the spin in any fixed direction (taken as the z-axis) is therefore not conserved and cannot be used to enumerate the polarization (spin) states of the moving particle."*

2) I think that the last statement in the following sentences is too strong and should be changed.

" In particular, while the S_m operators commute with each other and thus generate an $\mathbb{R}^3$ symmetry, the S_p on the rhs of Equation (25) shows that the L_m do not form a Lie subalgebra. Thus, $\mathbf{L}$ does not generate any symmetry at all."

I agree that the L_m do not form a Lie subalgebra, but one can still exponentiate a given L_m to generate a symmetry operator. That is, since L_m is self-adjoint, $\exp(-\mathrm{i}\theta L_m)$, for $\theta \in \mathbb{R}$, is still a unitary operator that maps photons to photons. Once can build eigenstates of such operator, and I do not think that one can exclude that some material system could possess such symmetry, that is, stay invariant after transformation with $\exp(-\mathrm{i}\theta L_m)$.

As explained in Section 5 of Reference [12], since $[L_m, S_m] = 0$, one can see $\exp(-\mathrm{i}\theta L_m)$ as the composition of a rotation and the transformation generated by S_m.

3) I find the following statement somewhat misleading:

" Massless fermions, known as Weyl fermions, are exceptionally rare, and have only been observed within the last decade in exotic materials."

It is my understanding that, for these quasi-particles, the linear dispersion relations that inspire the adjective "massless" do not have the same slope as a true massless particle in free space. In other words, the speed of light in such materials is smaller than c_0. Moreover, such dispersion relations are only approximately linear in the vicinity of a given point, and become more complicated when going away from such point.