Comments:

1. I disagree with this statement in the abstract: *"**Moreover, it has been shown that most of the proposed SAM and OAM operators do not satisfy the defining commutation relations of angular momentum operators and are thus not legitimate splittings**."* Consequently, I partially agree with the first half of the statement since the relativistic gauge-dependent SAM and OAM operators fulfill the cyclic commutation relations, and the gauge-independent operators do not. However, the second half of the statement about the splitting being non-legitimate is not fully correct in my opinion. The reasons for my concern are covered in the comments listed below.

2. One of the primary concerns addressed in this paper revolves around the angular momentum operators for massless bosons that do not follow the SO(3) commutation relations. This has been highlighted in the Introduction, in the statement: *"**However, most of these operators have been shown to either not be well defined or else not actually satisfy SO(3) commutation relations and are therefore not genuine angular momentum operators.**"* This directly questions the definition of Angular Momentum in the context of masssless bosons.

   The fundamental definition of the angular momentum operator comes from the fact that it is a generator of rotation for the corresponding symmetry group under analysis. In DOI: 10.1103/PhysRevA.26.3428 (1982), Lenstra and Mandel showed that the total angular momentum operator generates rotations for a general massless field, which is also highlighted in DOI: 10.1364/JOSAB.524752 (2024). **However, the derivations done by Lenstra and Mandel in 1982 do not imply that the commutation relations from SO(3) group is a necessary rule for angular momentum operators for the massless field.**

   Since the article deals with massless bosons, the little symmetry group ISO(2) should be the central theme for the angular momentum calculations, and not SO(3). The authors should consider modifying the sections to include the ISO(2) symmetry group, as opposed to SO(3) for these reasons.

3. ***Equations 11 and 12*** needs modifications, since these equations are valid for gauge-dependent angular momentum operators, and not for the gauge-independent case (please refer Paragraph 2 of Point 2 above). In fact, Reference (16) from the paper and DOI: 10.1364/JOSAB.524752 (2024) proves this statement.

4. This line in section 3 *"**The defining property of angular momentum operators is that they generate SO(3) symmetries**"*, which talks about massless bosons needs to be modified, since for massless bosons, the group ISO(2) should be the primary center of study, and not SO(3) which is the case for massive particles. This negates the need for reinforcing Equations 15 and 16. This directly leads to the modification of No-Go Theorem 1 in the context of massless bosons. The last line of Section 3 *"**The other set of operators,** (L_M^{obs}, L_M^{obs})**, are gauge invariant, but the** $L_{M,i}^{obs}$ **commute with***

each other rather than satisfy cyclic SO(3) relations and are therefore not angular momentum operators." should be modified for the same reason.

5. There are a couple typos in the paper:
    - "Masslesss" in Section 3 title
    - Some parts of the paper says SO(3), while others say $\mathfrak{so}(3)$