

## **Report on *Les Houches Lectures on Theoretical Ecology: high-dimensional models and extreme events*, by A. Altieri**

In these lecture notes, Ada Altieri explores the theoretical foundations of high-dimensional ecological systems using methods drawn from statistical physics and the study of disordered systems. The text begins with an in-depth analysis of MacArthur's Consumer-Resource (CR) model, a cornerstone of niche theory, which is reinterpreted through high-dimensional formulations. This allows for the investigation of feasibility and stability criteria, as well as steady-state solutions, which are treated within the framework of constraint satisfaction problems.

The author then demonstrates how the Generalized Lotka-Volterra (GLV) model can be derived from the CR model and proceeds to study the resulting dynamics using the cavity method in the absence of noise. This sets the stage for the next section, which extends the GLV equations to include stochastic effects. Here, both Itô and Stratonovich interpretations of noise are considered, and the replica method is applied to analyze the systems disordered free energy landscape. A key focus is placed on the interpretation of order parameters and the conditions under which ecosystems exhibit either a single stable equilibrium or multiple equilibria.

In the final section, the lecture notes present a derivation of the phase diagram under finite noise, emphasizing the zero-temperature limit of the replica symmetric solution. The work concludes with perspectives on several active research directions, including the application of Dynamical Mean-Field Theory to asymmetric and spatially structured interactions, the implications of heavy-tailed interaction distributions, and the incorporation of empirical data through data-driven modeling approaches.

These lecture notes are very well written and a pleasure to read. They will be extremely useful for anyone seeking a structured overview of theoretical ecology from a statistical physics perspective, and they offer a wealth of relevant references for those wishing to explore the field more deeply.